Numeration is the
arte to expresse and
rede all summes pro
posed, and is of two
sortis, for either it
thereth the palewe
a summe proposed,
ther els it expres
a summe conce
by tygures
ces due. Scho. Why? than me thin you
put a difference betweene the valu the
figures. M. Yea, so doo I. F the
one thyng, & the figures are a the
and that cometh par ely by of
figures, but chiefely o wherein
they be sette. S. Th n. we here
iii. thynges: the U gure, and the
Place. M. Euen yet adde Order
to theim, as the fi ieste marke
that there are but re vsed in
arithmetike, and of t e doth signi-
fie nothyng, which e an, o, and
is called priuately a ough all the
other sometyme be ykewise named. The o-
ther. ix. are called signifiyng figures, and be
thus figured.

Acyphar.

Figures.

5 6 7 8 9

is their value.

vj. vij. viij. ix.

marke, that euery fi-
gure

gure hath. ii. valewes: One alwaies certayne that it signifieth propr[illegible]ly, whiche it hath of his forme. And the other vncertayne, which he taketh of [illegible] place.

¶ A place is called the seate or roume that a figure stande [illegible]h in. And looke howe many figures ar writen in one summe, so many pla[illegible] that hole value. And the fyrst place must be called that, that is nexte to the right hande, and so reckenyng by order towarde the le[illegible] hande, so that that place is last, that is [illegible] to the lefte hande. As for example: if [illegible]de before you syxe men in a rowe, sy[illegible]yde, and you shuld tell them as thei sta[illegible]der, begynnynge with the man that were n[illegible] to your right hande, than he that were nexte hym, shulde be called the seconde, and so forthe to the farthest from your right hande, whiche is the. vi. and the last. a Place.

S. Syr I perce[illegible] you wel, so might I reckē letters or a[illegible]o[illegible]er thynge. As if I shulde write. viii. let[illegible]s after this order, a, b, c, d, e, f, g, h, no[illegible]uste I saie, h, is the firste, g. the. ii. f. [illegible]ii. e. the. iiii. d. the. v. c. the. vi. b. th[illegible] and a. the. viii.

M. That is well done. And after the same sorte vse hereafter, so that what I [illegible]are by one example, doe you expresse b[illegible] an other, and so I shall perceiue whether you vnderstande it or no. And so passe ouer [illegible]hynge till you perceiue it wel, and be experte[illegible]n.

[illegible]o.

S. Syr, I praie you, how many of these places be there in all? M. There is no certaine nūbre of them, but thei are sometimes more, and sometymes fewer, accordynge to the summe that is expressed. For so many as the figures are, so many are the place [illegible] and the last place is so called, not bicause it is the last of all other, but is the laste of the present summe, and it maie be the middell place in an other summe. S. Me semeth I perceiue this very well, as touchyng the or[illegible] of reckeuyng of the places. But as for [illegible]umbre of them you saie there is no cer[illegible]. Now there resteth to declare the va[illegible] the figures, by diuersitee of places [illegible] you called the valewe vncertaine. M. But fyrste let me heare whether you knowe perfectely the certaine valewe. S. Yes syr, as you wrote them, so I marked them. M. How write you then. v? S. B[illegible]s figure. 5. M. And how. vi? S. Thus. [illegible]. write these iii. noumbres eche by it [illegible], as I speake them. vii. iiii. iii. S. 7. [illegible]3. M. Howe write you these foure oth[illegible] i. ix. viii? S. Thus (I trowe) 2. 1. 6. 8. M. Nay, there you mysse: Loke on myn example agayn. S. Syr [illegible]th it is, I was to blame, I toke 6 for 9, but I will be warer hereafter. M. Now then take hede. these certaine valewes ever[illegible]gure representeth, when it is alone [illegible] without other figures ioined to hym.

Valewe vncertaine.

And

And also when it is in the first place, though many other doe folow as for example: This figure 9 is .ix. standyng now alone. S. How is he alone and standeth in the middell of so many letters? M. The letters are none of his felowes. And if you were in Fraunce in the mydde of M. Frenche menne, if there were no Englishe man with you, you wolde recken your selfe to bee alone. S. So it is. Then 9 without more figures of Awgrim, betokeneth .ix. what so euer other lettres be about it. M. Euen so, and so doeth it, if it be in the firste place ioined with other, how many so euer doe folowe, as in this example 3679, you se 9 in the firste place, and doeth betoken .ix. as he were alone. S. I perceiue that. And doeth not 7 stande in the seconde place, and betoken. lxx? and 6 in the thirde place and betoken. vi? And so 3 in the fourth place, and betoken. iii? M. Their places be as you haue saied, but their valewes are not so. For as in the fyrste place, euery figure betokeneth his owne valewe certaine onely, So in the seconde place euery figure betokeneth his owne valew certaine .x. times as in that example, 7 in the seconde place, is lxx. tymes. x. that is. lxx. And in the thirde place, euery figure betokeneth his owne valewe, a hundreth tymes, so that 6 in that place betokeneth. vi. hundred. And in the iiii. place, euery figure betokeneth his owne valewe,

3 7 6

valewe a. M. tymes, as in the foresayde number: 3. in the fourth place standeth for iii. M. And in the fyfth place, euery figure standeth for his owne valewe. x. M. tymes. And in the. vi. place a. C. M. tymes. And in the. vii. place, a. M. M. tymes. And in the. viii. place. x. M. M. tymes: so that euery place excedeth the former ten tymes. S. As thus, if I make this numbre at all aduentures, 90359684, here are. viii. places In the fyrst place is 4, and betokeneth but iiii. In the seconde place is 8, and betokeneth. x. tymes. viii. that is. lxxx. In the thyrde place is 6, and betokeneth. vi. C. In the fourthe place 9, is. ix. M. And 5 in the. v. place, is. x. M. tymes: [illegible] is. l. M. So 3 in the. vi. place, is a. C. M. tymes. 3. that is. CCC. M. Than 1 in the. vii. place, a M. M. And 9 in the. viii. place. x. M. M. tymes 9, that is. xc. M. M. But nowe I can not easyly nor quickely lede it in order. M. That shall you practise by this meanes. Fyrste put a pricke ouer the [illegible] the figure, and so ouer the. vii. And if [illegible] had so many, ouer the. x. xiii. xvi. and so foorth, styll leauynge two fygures betweene eche two pryckes. And those roumes betweene the pryckes, are called, ternaries. Than begyn at the laste pryche, and see howe many fygures [illegible] betweene hym, and the ende, whiche [illegible] not passe thre, reckenyng hym selfe

Ternary or Trinitee.

for

you hau[illegible] ready reckoned, to make them in a place [illegible]her, then thei shulde be, if he were awaie, and therfore now you shall lette hym go. And so do alwaies when he occupie the place nexte before any pricke, whiche is the last of that ternarie: and a cipher in the last place doeth nothyng. 8. Then I shall sale [illegible] 89. M. M. M. No, but go forthe. 8. 105. M. Now are all my prickes spent, and yet remaine 340, so I [illegible] valewe the. CCC. xl. onely. M. Now can you reckon after this sorte. And remembre, that euery suche roume so parted, is called a ternarie or [illegible]. Some doe parte suche great [illegible] with letters after this [illegible]

c a b c b a c b a c b a c b a

23086 4089 105340. In whiche example ye mue se that a, [illegible] the roume of your pricke. And some doeth parte the numbres with lynes after this fourme, 23086 4089 105340. where you se as many lynes, as you made prickes, and to [illegible] that the [illegible] doeth more [illegible] euery [illegible], accordyng as [illegible] be valewe [illegible] one denomina[illegible] syr, but if you shulde shewe [illegible] parted, I shulde take it for many [illegible] and not for one. M. Do [illegible] knowyng my meanyng. [illegible] set forth the numbre [illegible] selfe (for [illegible] of rec[illegible]

it with lines, wolde you forget wherfore you did it, & then take them for many numbres? S. No, I trowe not, but yet I doubte. M. Then vse that, that you lyke best, for all the thre waies is to one entent, saue (as I saied) that the lynes doeth more plainly distincte the denominations. S. What call you denominations? M. It is the laste valewe or name added to any summe. As when I saie: CC.xxii.poundes: poundes is the denomination: and like waies in saiyng: 25 men, men, is the denomination, and so of other. But in this places (that I spake of before) the last numbre of euery ternary, is the denomination of it. As of the first ternary, the denomination is vnitees: and of the second ternary, the denomination is thousandes: & of the third ternary, thousandthousandes, or myllions: of the. iiii, thousande thousand thousandes, or thousandmillions, and so forthe. S. And what shal I call the value of the. iii. figures that maie be pronounced before the denominators, as in saiynge: 203000000, that is, CC.iii.mylliõs, I receiue (by your wordes) that myllions is the denomination, but what shal I call the. CC. iii. ioined before the myllions? M. That is called the numeratour or valewer, and the hole summe that resulteth of them bothe, is called the summe, value or numbre. S. Now is there ought els to be lerned in Numeration

Denomination.

Numeratours

Summe.

racion? or els haue I learned it fully? M. I mought here shew you, who were the first inuentours of this arte and the reasons of all these thynges that I haue taught you, as why you shulde recken your order of places backeward, I meane from the right side towardes the left, with many other thinges touchyng the causes and reasons of it, but that will I reserue till ye haue lerned ouer al the practise of this arte, least I shulde trouble your witte with ouer many thynges at the first. But yet this must you marke, that Thre kyndes of numbre. there are thre kyndes of numbre: one called digettes: an other articles: and the thyrde myrte numbre. A dyget is any numbre vnder 10, as this: 1.2.3.4,5.6.7.8.9. And 10 with all other that mate be deuided into x. partes iuste, and nothyng remaine, are called articles, suche are 10. 20. 30. 40. 50. &c. 100. 200. &c. 1000. &c. And that numbre is called myrte that conteineth articles, or at the leaste one article, and a digette, as 12. 16. 19. 21. 38. 107. 1005, and so forthe. And for the more ease of vnderstanding and remembrance, marke this: The diget numbre is neuer written vo more then one figure, but the article & the myrte numbre are euer written vo more then one figure: & thus they differ, y the article hath euermore this cipher o in this first place, & y myrt numbre hath euer there some diget. S. By this la[illegible]des,

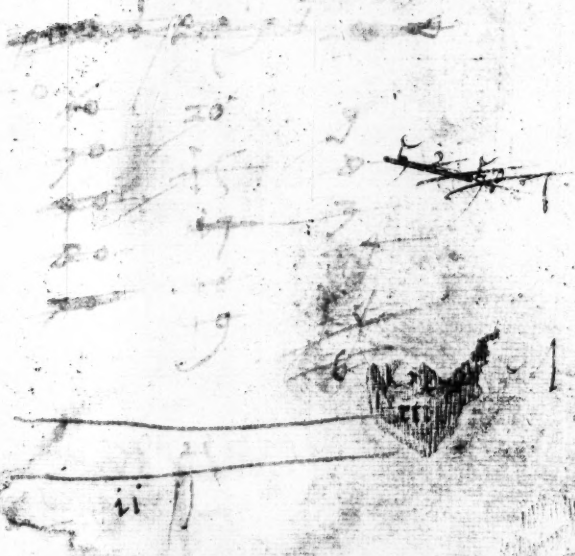
Digette.

Myrte.

Articles.

I perceiue it much better then I did before, & now (I thinke) I wil neuer misse to know those thre a sonder. M. If you remembre now all that I haue saied, you haue learned sufficiently this firste kynde of Arithmetike called Numeracion. Howbeit I will yet exhorte you now to remembre bothe this that I haue saied, and all that I shall saie, and to exercise your selfe in the practise of it: for rules without practise, is but a light knowledge: and practise it is, that maketh men perfecte and prompte in all thynges. And as you haue learned to gather the valewe of a summe proposed, so must you practise to expresse any summe with their dewe figures: as for a proue: How expres you this summe: fyue thousande, two hundred, fyfty and seauen? S. This troubleth me now, wheather I shulde begyn at the firste figure or at the laste, For reason (me thinketh) shulde cause me to begynne at the firste, and yet if I writ it as you spake it, I muste begyn at the last. M. When you knowe your places perfectly, you maie begyn where you lyste, but the more ease for your hande is, to begyn with the laste, that is to saie, as I did speake the. But for the more suretee a while you maie begyn with the first, repetig my wordes backwarde, thus: Seuen, fyfty, two hundred, fiue [illegible]sande, or els soundyng them all by their [illegible] or valower, as thus: Seuen, fiue,

two, fiue, for that waie is easyest. But than must ye looke well, whether there be any cyphar in your summe, that he maie bee set in his place, as if your laste valewer of youre summe (as you speake it) be aboue 9, than is there a cyphar in the fyrst place. And if it be a hundred or aboue, than is there. ii. cyphars, one in the fyrst place, and an other in the seconde, and so foorth. But because this thing is such that can not be set foorth with out many woordes, I thynke best here nowe at the ende of Numeration, to adde a table easy and ready for the fyrst exercise of it. Lo this is that table.



The ryght syde or hande.

The names of the digettes, valewes certayne, or valowers:

The denominatours of the places, or value vncertayne.	Nynes.	Tennes.	Hundredes.	Thousandes.	x. thousandes.	C. of thousãds.	Millions.	x. of millions.	C. of millions.	M. of millions.	x. M. of millions.
Nyne.	9	9	9	9	9	9	9	9	9	9	9
Leyght	8	8	8	8	8	8	8	8	8	8	8
Seuen	7	7	7	7	7	7	7	7	7	7	7
Syre.	6	6	6	6	6	6	6	6	6	6	6
Fyue.	5	5	5	5	5	5	5	5	5	5	5
Foure.	4	4	4	4	4	4	4	4	4	4	4
Thre.	3	3	3	3	3	3	3	3	3	3	3
Two.	2	2	2	2	2	2	2	2	2	2	2
One.	1	1	1	1	1	1	1	1	1	1	1
cyphar	0	0	0	0	0	0	0	0	0	0	0
The order of the places.	Fyrste.	Second.	Thyrde.	Fourthe	Fyfth.	Syxte	Seuenth	Eighteth	Nynthe.	Tenthe.	Eleuenth.

The lefte syde or hande.

¶ This table (as you maie see) hath aleuen places, and in eche of them are set all the digettes, whose certayne valewe is written on the [illegible] hande of the table, and the valewe vnce[illegible] on the lefte hande. So that by

thi

this table you maie learne both howe to expresse any numbre that you lyste (yf that it excede not.xi. places) that is to saie. lxxxx. thousande myllious, and so maie you by the helpe of it, valewe all summes proposed vnder the sayde numbre, as for exaumple: the summe that I proposed before, whiche was fiue thousand, two hundred, fifty and seuen, yf you will expresse it, take the fyrst numbre (as I speake it) whiche is v. M. whose valower or certain valow is.v, & his vncertayn valowe or denominacion is. M, Fyrste you shall seke at the ryght hande, the valewer. v. Than seeke alonge vnder the title of denominacion, towarde the lefte hande, tyll ye fynde thousandes, and vnder it ryght at the foote of the table, is the numbre of the place that is the fourthe, wherein you muste write youre digite or valower, 5: Than come to the seconde parte of the numbre.ii.hundred, whose valower is 2, and denominacion. C: Seeke. ij. at the ryghte hande of the table, and go along vnder the denominacions, towarde the lefte hande, tyll you come vnder C: Than loke to the foote of the table, and there shall you see the numbre of the place, that is to saye.iij, wherin you must set your digite 2: Than doo so by youre other two numbres that remayne, and you shall fynde fyue in the seconde place for your yoke, and 7 in the firste place for your [illegible] thus

maie you doe with other numbres. S. Maister I thanke you hartily, I perceaue you seke to instructe me moste plainly and brefely, and not to hyde your knowledge with subtill wordes, as many doe: For this rule is so plaine that I can desire it no plainer. And though it seme somewhat longe, yet I perceaue it a sure waie. M. So is it, and though it be longe, yet it is neither to long, neither to plaine for yonge learners that lacke practise, for this table is in stede of a teacher, to them that lacke one. But now I truste I haue saied enough of Numeration, whiche, after you haue well practised, then maie you learne forthe. S. Syr in numeration I haue well practised, and am ready to learne forthe. M. That is well. But what shulde you nexte learne can you tell? S. I remembre you saied, that Addition was next. M. Euen so, and what that is, muste you firste knowe.

ADDITION.

Addition is the reduction and bringyng of two sũmes or more into one: as if I haue 160 bokes in the latine tongue, and 136 in the greke tongue, and woulde knowe how many they bee in all. I muste

muste write those. ij. numbres, one ouer an other, writyng the greatest numbre highest, so that the firste figure of the one, bee vnder the firste figure of the other. And the second vnder the seconde, and so forthe in order. When you haue so done, draw vnder them a straite line, then wil they stand thus

160
136

Now begyn at the fyrste places towatde the ryght hande alwaies, and put togither the. ij. first figures of those two summes, and loke what cometh of thē, write vnder them, right vnder the lyne: As in saiyng, 6 and 0 is 6: Wryte 6 vnder 6, as thus.

160
136
6

And then go to the second figures, and doe lyke waies: as in saiyng, 3 and 6 is 9: write 9 vnder 6 and 3, as here you se.

160
136
96

And lyke waies do you with the figures, that ben in the third place, saiyng, 1 and 1 be 2: write 2 vnder them, & then wil your hole sūme apere thus.

160
136
296

So that nowe you see, that 160 and 136, doe make in all, 296. S What? this is veray easy to doe, me thinketh I can doe it euen sith. There came through Cheapes syde, ij. droues of cattelle, in the firste was 848 shepe. [illegible]d in the seconde was 186 rother beastes. [illegible]hose two summes I muste write [illegible]s yo[illegible] [illegible]ught me thus,

848 Than if I put the. ij. first figures
186 togither, saiyng: 6 and 8, thei make
14, That must I write vnder 6 and 8, thus
848 M. Not so, and here ar you twise
186 deceaued. Fyrst, in goynge about e
14 to adde togither. ij. summes of sun
dry thynges, whiche you ought not to doo,
except you seke onely the numbre of theim &
care not for the thynges. For the sūme that
shuld result of that addition, shuld be a sūme
nother of shepe, nor other beastes, but a con-
fused summe of bothe. How be it, somtymes
you shall haue summes of dyuers denomina
cions to be added, of whiche I will tell you
anone: But fyrst I will shew you, where you
were deceaued in an other poynte. and that
was in writynge 14, which came of 6 and
8, vnder 6 and 8, which is vnpossible: For
howe can two figures of two places, be wri-
ten vnder one figure and one place? S. Tru-
the it is: but yet I dyd so vnderstande you.
M. I saied in deede that you shoulde write
that vnder theim, that dyd resulte of theim
bothe togyther, whyche sayinge is alwaies
trewe, if that summe do not excede a digite.
But if it be a mixt numbre, than muste you
write the digite of it vnder your figures, as
I ha[illegible] sayde before: But and if it be an ar-
ticle, [illegible] write o vnder theim, and keepe
the art[illegible] in y[illegible]ur mynd. And therfore whā
you hau[illegible] your seconde figures, which
occupie

occupie the place of tennes, you shall putte that 1 therto, which you kept in your mind: for thoughe it were 10 in deede, yet in that place it is but as 1, because that euery 1 of that place is ten, for it is the place of tēnes. a Place.
And in lyke maner, yf you haue in the seconde place so great a numbre, that it amoūteth aboue 9, than write the digites, and reserue the article in your mynde, euer addyng it to the next place folowyng: and so of all other places, howe many so euer you haue. And if you haue a myxt numbre, whan you haue added your laste figures, than write the digite vnder the laste figures, and the article in the next place beyonde theim: So shal your numbre resulting of addition, haue one place more than the numbres, whiche you shulde adde togither. S. Nowe doo I perceaue you, and the reason of this is (as I vnderstande) because that no one place can conteyne aboue 9, whiche is the greatest figure that is. And than all tennes or articles must be put to the nexte place folowyng: for euery place (as I maiese) excedeth the other place next before hym by. x. Now (if it please you) I will returne to my exaumple of cattell. But I remembre you sayd I might not adde summes of sundry thynges [illegible]gyther, and that myghte I see by reason [illegible]aister. Trouthe it is, yf you seke the s[illegible]e of any thigs: but if you only seke [illegible] haue no

respect

respecte to the thyng, than were it better to name the summe onely without any thyng: as in saiyng 848, without namyng shepe or any thyng els. And like waies 186 namyng nothyng. Nowe lette me see, howe can you adde those two summes. Scho. I must firste set theim, so that the two first figures stande one ouer an other, and so the other, eche one ouer his felow of the same place, than draw a lyne vnder them bothe. And so lyke waies of other figures, settyng all waies the greatest numbre hyghest, thus.

848
186

Than must I adde 6 to 8, whiche maketh 14, that is a myxte numbre, therfore muste I take the digett, which is 4, and write it vnder 6 and 8, keepyng the article 1 in my mind thus.

848
186
4

Than doo I come to the secoude figures, addyng theym togyther, saiyng: 8 and 4 make 12, to whiche I put the 1 reserued in my mynde, and that maketh 13, of whiche numbre I write the digite 3 vnder 8 and 4, and keepe the article in my mynde, thus.

848
186
34

Than come I to the thyrd figures, saying: 1 and 8 make 9, and 1 in my mynde maketh 10. Syr shall I wryte the cypher vnder 1 and 8? M. Yea. Scho. Than [illegible] I write the cypher vnder 1 and 8, and [illegible] the article in my mynde. Maist. What ne[illegible] sleeyng there foloweth no more

more figures? S. Syr I had forgotten, but I wyll remembre better hereafter. Than seyng I am come to the laste figures, I muste wryte the cyphar vnder them, and the article in a farther place after the cyphar, thus.

848
186
1034

Maist. So now you se, that of 848 and 186 added togither, there amounteth 1034. S. Nowe I thynke I am perfect in Addition. M. That will I proue by this exaumple. There are. ii. armies of souldiours, in the one are 106800, and in the other 9400: Howe many are there in bothe armies, saie you? S. Fyrst I set them one ouer an other, begynnyng with the fyrst noumbres at the ryght hande, thus.

106800
9400

But the nether numbre wyll not matche the ouer numbre. M. That forceth not. S. Than do I adde o to o, and there amounteth o, that must I wrte vnder the fyrst place, thus.

106800
9400
o

M. Well sayd. S. Than lyke waies in the seconde place I adde o to o, and there ariseth o, whyche I wrte vnder the second place, thus

106800
9400
00

Than I come to the thyrd place, saiyng: 4 and 8 make 12, of whyche I wrte the digite 2. and kepe the article 1 in my mynde, thus.

106800
9400
200

Than adde I 9 to 6, which maketh 15, to that I adde the arti-

cle. 1. that was in my mynde, and it is 16, I writ 6 vnder 6 and 9, and kepe 1 in my mynde, thus.

106800
94 00
6 200

M. Why do you not write bothe figures, seyng you are come to the laste couple of numbres? S. Nay, reason sheweth me that I muste adde that article that is in my mynde, vnto the nexte figure of the ouer summe, though there be no more in the nether summe. M. That is wel considered, then doe so. S. Then saie I, 0 in the ouer summe, and 1 in my mynd, make 1: that I write vnder 0: The followeth there 1 in the ouer summe, whiche hath none to be added to it, for there is none in the nether summe, nor yet in my mynde, therefore I thinke I muste write that euen as it is. M. Yea. S. Then doeth my hole summe appere thus.

106800
94 00
116200

M. If you marke this, you haue learned perfectly the commen addition of all summes whiche are of one denomination, so that you obserue this also, that in Addition you must haue ii. numbres at the leaste, or els how can you saie that you doe adde? And euer lette the greatest numbre be written hygheſt, for that is the beste waie, though it be not necessary. And [illegible]et not this: that if you haue many numbres to adde to gether, you shall haue oftentymes an article of a greater valew the

10,

10: sometymes 20, sometimes 30, sometimes more, yea peraduenture 100: Therefore as you did with the article 10, so do with them, reseruyng them in your mynde, and addyng to the numbre nexte folowynge so many, as their valewer or valewe certaine is, that is to sate: 2 for 20, 3 for 30, and so forthe of other. But if the article be 100, then muste you not adde the article to the nexte figures folowynge, but to the thirde figures from them: as I will shewe you anone by example. And if it chaunce the nombre to be such, that it doe comprehende two sundry articles (that is one of tennes, and an other of hundredes) then muste you reserue them bothe in your mynde, and adde the article of tennes to the figures that folow nexte, and the article of hundredes to the figure of the thyrde place from thence. Now take this example for all. I wolde adde these xiii. summes in one, which I set after this maner. Then doe I begyn & gether the summe of the firste figures which commeth to 107, For first I take 9 there .x. times, & that is 90: then 9 and 8 is 17, that is in al 107: Of which sume I write the 7 vnder the first figures, and then haue I an article of 100 in my mynde, whyche other I muste

4	8 8 9
4	5 9 9
2	2 9 9
3	6 9 9
2	3 9 9
4	0 9 0
1	0 9 9
3	1 9 8
	2 9 9
	6 9 9
	8 9 9
	1 9 9
1	8 9

I must kepe in my mynde, till I come to the thyrde figures, whiche are in the roumes of hundredes: or els I maie for feare of forgettyng, wrıte one vnder the thyrde rewe of figures, makyng two lynes as you see here dooen. And than muste I wrıte the digites vnder the lowest lyne, and this is the surest waie whan the summe is so greate, that the addition of one rowe passeth 100. Whan I haue so done, I must than come to the secõd rowe of figures, and adde theim togyther, whiche doothe make 115, of whiche summe I wrıte the digite 5 vnder the same seconde row, and then haue I an article remaynyng of two figures, of which the one (that standeth for 10) must be added to the second, or nexte place after theim, that I dyd laste adde. And the other (that standeth for 100) must be added to the thyrde place from thense. S. That is to saye, the fourth place frō the first line or rowe of figures. M. Euen so. And than wyll the summe appeere thus. Than adde the thyrd rowe of figures with the two vnitees betwene the lynes: and the summe amounteth to 50, of which I wrıte the cyphre vnder

	4	8	8	9
	4	5	9	9
	2	2	9	9
	3	6	9	9
	2	3	9	9
	4	0	9	0
	1	0	9	9
	3	1	9	8
		2	9	9
		6	9	9
		8	9	9
		4	9	9
		3	8	9
	1	1		
		5	1	
2	9	0	5	7

vnder the same thyrde row, and the 5 vnder the nexte fignres toward the left hand. Thē I adde the fygures of the fourth rowe with the 1 and 5, that are vnder them betwene the two lynes, and thei make 29: then write the 9 (that is the digette) vnder the fourth place and the 2, that is the article beyonde it towarde the lefte hande. So those summes do make 29057. S. This semeth somewhat harde, by the reason of so many numbres together: How be it I thinke if I dooe often proue euen with this same example, I shall be able to do so, shortly, with any other sūme. M. So shal you: For it is often practise that maketh a man quicke and rype in all thinges: But because of suche great summes, in which there maie chaunce to be some errour I wil teache you, how you shall proue whether you haue done well or no. S. That were a great helpe and ease. M. Begyn first with the highest numbre, and then al the other orderly, and adde them together, not hauyng regarde to their places, but as though they were all vnitees, and still as your noumbre encreaseth aboue 9, caste awaie 9: Then go foorthe, euer castyng awaie 9, as often as it amounteth thereto: And so do til you haue gone ouer all the numbres that you intended first to adde, and what so euer remaineth after suche addicion, and castyng swaie of 9, write it in sōme voide place, bis he ende

Proofe of the first parte of addition

of a lyne, for ẏ better remembraunce. And then putte together the figures that resulte of the addition, styll castyng awaie 9 also: And then that, that remaineth, write at the other ende of that line, and if those two bee lyke, then haue you well done, but if thei be vnlyke, then haue you missed. As for example in this present summes: The first figure of the oouer lyne is 9, lette hym go: then 8 and 8 is 16, take awaie 9 and there remaineth 7, adde to it 4 that foloweth, and that maketh 11, from whiche if you take 9 there resteth 2. Then come to the nexte rowe, whose firste and seconde numbre are 9, therfore ouerpasse them bothe and take the 5 to the 2, whiche did remaine in the firste rowe, that maketh 7, put therto the 4 folowyng, that maketh 11, thence take 9, and there remaineth 2. Then go to the third lyne, whose two fyrst numbres you maie let passe because thei are nynes: then take the two 2, whiche with the other 2 that remained in the second rowe, make 6. Then go to the fourthe rowe whose two fyrste numbres let go, and take the 6 to the 6 that remayned, and that maketh 12: take awaye 9, and there resteth 3, whiche with the 3 that is nexte, maketh 6: And so go through all the other noumbres, you shall synde that there remaineth 5, after you taste awaye 9 as often as you synde it there are wryte 5 at one ende of a lyne

in

in a voide place thus. 5 ——— Then gether all the figures of the totall ſumme whyche is vnder the loweſte lyne, and caſte awaye 9 as often as you fynde it, as thus: 7 and 5 make 12, take awaie 9 and there reſteth 3, to that if you adde the 2 that is laſte (for you maye let go ẏ 9) then doeth it make 5, whyche you muſte wryte at the other ende of the line, that you made in the voide place, and it wyll bee thus, 5 ——— 5. And then you ſe that thoſe two fygures bee lyke, whereby you maie knowe that you haue doen well, and ſo maye you proue in any other. Scho. If it pleaſe you I will proue in an other ſumme. Maiſt. With a good will. Scho. Then wil I take one of youre former examples, whyche was this.

Fyrſt in the higheſt lyne 8 and	106800
6 make 14, then 9 taken awaie	9400
remaine 5, to whyche I addde	116200

the 1 that foloweth, and that maketh 6. Then come I to the ſecond lyne, where I fynde firſt 4, whyche with 6 maketh 10, from that I take 9, & there reſteth 1, the next figure is 9, and therfore I let him alone: ſo I fynde 1 remaining, which I ſet at the ende of a line thus 1 ——— Then I come to the totall ſumme, and there fynde ẏ all the figures put together make 16 frō which I take 9, & there reſteth 1 alſo which I put at ẏ other ende of ẏ line thus ——— 1

And because thei be like, I knowe that I haue well added. M. So you knowe nowe bothe how to adde two summes or more together: And also how to proue whether you haue done well or no: whiche thyng also you maie doe by Subtraction. But because you can not yet skyll of it, I wyl lette that passe till anon, and will teache you now howe to adde summes of diuers denominations: whiche thyng can neuer be, but when the one denomination is suche, that it conteineth the other certeine tymes. And yet you shall adde them to the other, not after this sort as you bydde them that were of one denomination: but after suche a sorte, as I will now shewe you, that is to saie: If you haue a summe of diuers denominations, then loke that ye set euery denomination by hym selfe with sume note or figure of his denommination, as they be wonte to be written: Then write your other summes so vnder that firste, that euery one be set vnder the other of the same denominations. As for an example: If your denominations be poundes, shyllynges, and pens, write poundes vnder poundes, shyllynges vnder shyllynges, and pens vnder penys: And so of shillynges vnder pens, nor penys vnder poũdes. S. Now that you haue spoken it, me thinketh it neded not to warne me of it, for it were against reason so to confounde summes: but yet if you had not spoken

ken of it, peraduenture I shoulde haue ben deceaued in it. M. If you dooe saie it is so plaine, I will speake no moze of it, but with an example make the matter to appere euydently. Fyrste one man oweth me 22 li. 6 s. 8 d. Another oweth me 5 li. 16 s. 6 d. And an other oweth me 4 li. 3. s. I wolde knowe what this is all to gether. Therfoze must I firste set downe my greatest summe, & then the other, euery one vnder his denominacion, agreyng to the greatest summe, as thus.

	li.	s.	d.
Then muste I begynne at the	22	6	8
smallest numbres whiche muste	5	16	6
alwaies be sette nexte the right	4	3	
hande, and adde them together.			

And if the summe of them will make one of the nexte denominacion, then muste I kepe it in my mynde, tyll I come to that place; or elsfoz moze easinesse, write it vnder that place betwene the double line, & vnder that firste place muste I note the residewe, if there remaine any of the same denominacion, but if there remaine none, then nede I to write vnder it nothyng. And this is all that you muste marke in this addition: foz all other thinges are lyke to the other maner of addicion, before mencioned. Therfoze the cheifeste pointe of this addicion is, to know the valewes of commen coine, & rated summes: As how many shillynges be in

a pound, how many penies in a shillyng. Of which and of other lyke thynges, I wyll instruct you hereafter, in teachyng of Reduction. But now I may not disturbe your witte from the thyng that we are about: Therfore let vs returne to that former example, whiche I proposed of .iij. letters, which sithes whan I had set orderly, thei stode thus with a double lyne vnder them.

li.	s.	d.
22	6	8
5	16	6
4	3	

Then to adde the vnto one summe, muste I begyn at the ryght hand where the smallest denomination is, and adde them togither first, saiynge 6 & 8 make 14: now seyng these 14 are penies, & that 12 d. make one shillyng, whiche is the next valewer, I take away 12 from 14, and there resteth 2, which I wyte vnder the penies, and for the other 12, whiche make 1 shillyng, I wyte 1 vnder the title of shillynges, thus.

li.	s.	d.
22	6	8
5	16	6
4	3	
	1	
		2

Then doo I adde all the shillynges togyther, and find them 25, to which I adde the 1 betweene the two lines, & that maketh 26, but because that 20 s doo make 1 li, I take awaye 20 from 26, and for that 20 I wyte 1 vnder the pounds betwene the two lynes, and the other 6 that remaineth, I wyte vnder the shillynges: thus.

li. s.

Then come I to ỹ poundes addyng theim all togither, and fynde theym to bee 31, therto I adde the 1 betwene the. ij. lines, & that maketh 32, whiche ſumme I write downe hole, bicauſe there is no greater denomination: And than my holle ſume appereth thus.

li.	s.	d.
22	6	8
5	16	6
4	3	
1	1	
	6	2

So is my totall ſumme 32 li, 6 s, 2 d. And this maie you proue m an other lyke ſumme. S. The wyl I caſt the holecharge of one monethes cõmons at Oxforde with battelyng alſo. M. Goto, let me ſee howe you can doo. S. One wekes cõmons was 11 d ob. q, and my battelyng that weke was 2 d q, q. The ſeconde weekes cõmens was 12 d, and my batlyng 3 d. The thyrd wekes cõmons 10 d ob, & my batlyng 2 d, q, c. The fourth wekes cõmons 11 d q, & my batlyng 1 d, ob. c. Theſe 4 ſũmes wold I adde vnto one hole ſũme, & therfore I will ſet them one ouer an other, thus. But I had forgotẽ, I ſhuld haue ſet the greateſt ſũme higher. M. So is it cõmonly beſt, howbeit here it forceth not, & in ſuche ſũms as this is, that go by order of wekes, daies, or yeres it is

11	ob	q
2		q. q
12		
3		
10	ob.	
2		q. c
11	q.	
1	ob.	c

it is better to keepe that order than to alter them, and to set the greatest numbre highest, for that serueth for suche summes as go not by order. S. Than yf I haue sette them wel ynough, I will begyn to adde theim thus. Fyrste of the smallest balewers at the ryghe hande, which are called ceys, I synde 2, and seyng that 2 ceys doo make one q, I wyll write nothyng vnder the ceys, but wil write 1 q, for 2 ceys vnder the kewes betwene the lynes, after this maner. Than come I to the next balewers, where I fynd 2 q, and to them I adde the q, that is betwene the lynes, and so are they 3 q, but because 2 q maketh one q̄, I write one q̄ vnder the farthynges betweene the lynes, and the q that remayneth, must I write benethe the nethermost lyne, vnder the kewes, thus.

Than com I to the farthynges where I fynde 3, and the other q̄ that is betwene the lynes maketh 4 q̄: And because 4 q̄ make iuste 1 ō, I shall write nothyng vnder the farthynges, but muste write 1 vnder the pens betwene

ō.	
11	ob. q̄.
2	q̄. q.
12	
3	
10	ob.
2	q c.
11	q̄
1	ob. c
1	q
1	
ō.	
11	ob. q̄.
2	q̄. q.
12	
3	
10	ob.
2	q. c.
11	q̄.
1	ob. c.
	q̄. q.
	q.

tweene the lynes. Than muste I adde the halfepence togither, of whiche there are 3, but seyng that 2 ob. make 1 d, I must write 1 vnder the pēs betwene the lines: but how shall I doo it, for there is 1 already? Mai. Haue you forgotten how I dyd in addition of the greate summe before? you muste set it vnder the other, so shall they both stande for 2: for if you shuld set it before or behynd the other, they shulde make 11. S. I remembre it now, and I perceaue the reason. Than I will write 1 ob. vnder the halfpens, and for the other two halfepens whyche make 1 d, I write 1 vnder the pens: Than come I to the pens, and fynde that there are of theim 52, than put I to theim the 2 betweene the lynes, and that maketh 54, which amounteth to 4 ß. 6. d, the 6 d. I must write vnder the pens, and the 4. ß. I muste sette (I suppose) farther towarde the left hand by them selfe. M. Euen so. S. Than appereth al my additiō thus. And the sum is 4. ß. 6 d. ob. q. M. Now haue you don this wel. But tel me, why dyd you write kewe, cee, thus q, c, and not rather thus qc, as the fashion is. Schol. Because I thought it was the best way

	d.		
	1 1	ob.	q.
	2		q. q.
	1 2		
	3		
	1 0	ob.	
	2		q. c.
	1 1		q.
	1	ob.	
	1		q.
ß	1		
4	6	ob.	q.

waie for dew gathering of euery denomination by him self. M. So was it in dede. Wel, now can you tell how to proue this addition, and such other like of diuers denominations, and to trie whether you haue doon wel or no? S. I would I could. M. That shall you doo by this meanes: Fyrste as you dyd begyn to adde, so recken agayne euery denomination by it selfe, and when you finde so many small, that doo make any other denomination, let theim go, and kepe in mynde onely the residew that wyll make no greatter denomination, and loke whether there be any suche lyke valewe vnder the nether lyne, and if there be, you haue wel doon, and so go forth frome one denomination to an other, vnto the ende. But here must you note, that in gathering of the summes you must recken those figures that are written betweene the lines with theim that are written aboue theim, as for an example. I will examine that summe that I dydde laste adde, whiche stode thus.

	li.	s.	d.
	22	6	8
	5	16	6
	4	3	
	1	1	
	32	6	2

Fyrst I find 6 and 8 whiche maketh 14, from whiche I take 12, bicause it maketh one of the nexte denomination, and there remaineth 2, and vnder that place I see a lyke figure, therfore I knowe that to be wel done. Than come I to the s, where I finde 1, 3, 16,

16, & 6, ỹ maketh 26, I cast awaie 20, for they make an other denomination, ỹ is to sai, poũdes, & the 6 which remaineth is like to the 6 ỹ is writẽ vnder thẽ beneth ỹ lowest line, therfore ỹ is wel done also. And thense I go to poundes, where I fynd 1, 4, 5, 22, ỹ is 32, to whiche sume agreeth an other lyke vnder it, therfore I iudge all well done. S. I perceaue reason in this probacion, nowe wyll I attempt the same in the summe that I dyd adde, whiche when I had ended adding, stode thus.

	℔.		
Fyrst amongest the cees	1 1	ob.	q̃.
I fynde but 2, whyche	2		q̃. q.
make 1 q, euen, therfore	1 2		
there muste nothyng bee	ʒ		
vnder the lyne for them:	1 0	ob.	
and amongest the kews	2		q. c.
I fynde 3, of whiche 2,	2 1		q̃.
make 1 q̃, therfore I let	1	ob.	c.
them go, and the one q,	1		q. q.
that is lefte, hath an o-	1		
ther like vnder his place	℥ 1		
therfore that is wel don	4 6	ob.	q.

Thã the farthings ar iust 4, which make 1 ℔, & therfore I let thẽ go. Amongest the halfe pẽs ther is one odde, for 2 make I cast away, bicause thei make 1 ℔, & vnto it answereth a like sum vnder it. The pẽs ar 54, fro which I take away 48, ỹ make 4 s̃, & ỹ 6 remainig agre to a like figure set vnder thẽ. & last of al remaineth ỹ 4 s̃, which ỹ abiected pẽs dyd

make

so I perceaue that I haue well doone. Nowe this wyll I not forget, but will this examination serue in all addition: Mayster. It seruethe for all addition of sundrye denominations, yf the addition bee made with two lynes (as these were) elles it wyll not serue, because that those summes, whiche ar here added betwene the lynes in addition by one lyne, are vnderstande and not written, but I let that waies passe, because as it is common, so is it more deceauable than this waies, namely if a mans memory be either dulle, other troubled. Scholar. Yet it were good to knowe that waies also. Mai. If you desyre to knowe it, this it is in few wordes: Doo euery thynge as you dyd in this sorte, saue that where you made here two lynes, you shall make there but one: and those summes that you dydde here write betwene the lynes, you muste keepe in youre memorye, and adde theim (as you dyd here) eche one whan you come to his place. S. Than they differe not but in this, that this additiō with two lynes leaueth nothyng to memorye, but writeth downe all: and the other waie commytteth certayne numbres to memorie, as you taught me in the fyrste examples of addition of smalle summes of one denomination. But what if a man vse it (as you saie men doō commonly) howe shall it be examined? M. Seynge you are so desyrous of it

I will

I wyll shewe bothe an example of the addi-
tion, and also the maner to examine it. I
propose these. iij. summes to be added, and I gather fyrst the pens, as I dyd in the other sorte, and I find

li.	s.	d.
12	8	9
6	7	3
3	6	8

of theim 8, 3, 9, that is 20, of which summe I bate awaie 12, whiche make 1 s, & kepe that 1 in my mynde, and the reste that is 8, I write vnder the pence. Than doo I adde the shyllynges togither, and fynde of theim 6, 7, 8, that is 21, whereof I bate 20, that make 1 li, whiche I kepe in mynde: and to the other 1 that remaineth, I adde that one that came of the pens, and was in my mynde, whyche make 2, & theim I write vnder the shyllynges.

li.	s.	d.
12	8	9
6	7	3
3	6	8
22	2	8

Than doo I recken the poundes togyther 3, 6, 12, that is 21, and to theim I adde the 1 in my mynde that remayned of the shillynges, whiche make 22, them doo I write vnder the poundes: and than my summe totall appereth to be. 22. li. 2. s. 8. d. Nowe to examin this summe (and all lyke) you shall doo thus. Fyrste begyn at the leste hande with the poundes, and take from theim that are aboue the lyne, 9 as often as you can, than that, that remayneth, shall you double, than ioyne it with the shillynges

proufe

lynges, and take awaie 9 from that as often as you can, and what so euer remaineth, you shall take for it 3 tymes so much and put to the pennies: then take from all that sume 9 as often as you can, & what so remaineth after you haue withdrawē 9 as often as you cā, write ỳ at ỳ end of a line, as I taught you in an other addition. And then come to the summe vnder the line, begynnyng with the poundes, and doe euen as you did with the summes abone the line, tyl you come to your pennies, and if the figure of the summe that remaineth after casting awaie 9 (as often as you can) do agree with the other that remained before of the other summe, which you dyd write at the ende of the line, than haue you done wel, els not: and for an example I will examine that last summe whiche was thus.

li.	s.	d.
12	8	9
6	7	3
3	6	8
22	2	8

Fyrste I shall begyn at the lefte hande with the poundes, puttyng them together, whyche make 22, in whiche summe I fynde 9 twise, (for twyse 9 is 18) that I deducte and there remaineth 3, ỳ 3 must I double (as I saied) because it is the remainer of the poundes, and it will be 6: Then gether I the summe of the shyllinges, which is 27, to the which I adde the forsaied 6, and then is it 27, wherin I find 9 thre tymes, and there remaineth nothing

This

This remainer shulde I take iij. times: but iij. tymes nothyng, is nothyng: therfore in this place is there nothyng lefte to be added to the pennies, therfore muste I take the summe of pennies alone, whiche is 20, from thence if I take 9 twise, there remaineth but 2, whiche I put at the ende of a lyne, thus.

2 ——— Then I come to the pounds of the vnder numbre or totall summe, and ther I fynde 22, from whiche I take awaie 9 twyse, & there remaineth 4, that 4 I double, and it is 8, then doe I adde that 8 to the shyllynges, and it maketh 10, from whiche I withdrawe 9, and there resteth 1, then do I take that 1 thre tymes, and it maketh 3, whiche I adde to the 8 ō, and it maketh 11, from which if I bate 9, there resteth 2, whiche is equal to the numbre noted at the ende of the line, & therby I perceaue that I haue done well. S. But I doe not se the reason of this. M. No, no more do you of many thinges els, but hereafter will I shewe you the reasons of all Arithmeticall operacions, for this I iudge to be the best trade of teachyng firste by summe breue preceptes to enstructe a learner somewhat in the vse of the arte, before he learne the reasons of the arte, and then maie you afterwarde more soner make hym to perceaue the reasons: for harde it is for to occupye a yonge learned witte with bothe the arte & the reasons of it all at ones:

how

how be it he shall neuer be cunnyng in dede in an arte, that knoweth not the reason of euery thyng touchyng it. S. Yet at the least I praie you show me why did you write your numbre that remaineth (after you had withdrawen all the lynes) at the ende of a lyne? for I sawe no reasō why that lyne did serue. M. Dyd you euer marke a carpentar when he wrought? S. Yea many tymes. M. And haue you not sene hym whan he hath taken measure of a borde, that he hath pricked it, and hath with a knitche of his hande drawen a lyne from the pricke that he made? S. Yes I haue marked that, & haue sene some make. iii. or iiii. lynes by the pricke, & some also haue I sene make a crosse by it, but that I perceaued was for ỹ easy findyng of their pricke. M. And euen so is this lyne, for the easy fyndynge of your remainer, and therfore some do make a crosse thus.

And set the one remainer aboue the crosse, and the other vnder the nether parte of the crosse, as if I shulde set my two remainers thus.

another proufe

But there is an other sorte of proue of addicion, to which the crosse serueth more meter. And that is, when the addicion is of diuers denominacions, and I wouldeexamine euery denominacion by it selfe, whiche waies though it be not muche vnlyke to the firste proue, that I brought of

fyche

ſuche dyuers ſummes, yet I will declare it, leaſt you ſhuld think that I wold hide it frō you: you muſte make ſo many lynes in your croſſe, as you haue ſundry denominations: as if you haue but 2 denominations, than you maie make it thus: that the ouer parte and the nether parte maie ſerue for one denomination, and the two ſides for the other. And yf you haue 3 denominations, as poundes, ſhyllinges, and penies, than muſt you make thre lynes, thus. The vpright lyne maie ſerue for the poundes, and the highest thwart lyne for ſhillyngs, and the lowest for pēnes, as for exaumple, I will take a ſumme thus added, for the proofe of the whithe, bicauſe it conteyneth thre denominations, I muſt make a croſſe of 3 lynes thus.

li.	ſ.	d.
16	12	5
12	8	1
9	2	7
38	3	1

Than I recken firſt at the right hand the penies 7, 1, 5, make 13, from which I take 12 for the next denomination, that is to ſay a ſhillyng, and there reſteth 1, which I muſt write at one ende of the nether thwart line. Than I gather the ſumme of the ſhillynges 2, 8, 12, whiche make 22, to them I put 1 that I toke of the penies, and that maketh

23, from those I take 20, the quantitee of the next greatter denomination, that is to saie a pounde, and there resteth 3, which I write at the ende of the ouerthwarte lyne. Than I adde togither the poundes, 9, 12, 16, whiche make 37, to them I adde the 1 that came of the shillynges, and than there is 38, wherin I fynd 4 tymes 9 and 2 ouer, that 2 I write on the vpright lyne. Than I come to the totall summe, and examine it begynnynge at the penies, where I fynde but 1, and can not take 9 from hym, therfore I set hym at the other ende of the nether thwart line. Than come I to the shillynges where I fynd onely 3, which because it is lesse than 9, I set it at the other ende of the line of shillynges, that is the ouermoste thwarte lyne. Than of the 38 li, I take oute 4 tymes 9, which is 36, & there remaineth 2, whiche I write vnder the vpright lyne, then I consider euery numbre, comparing it to the numbre that is against it, and because I fynd them to be euer one like his matche, I know that I haue wel don. S. This crosse I perceaue doothe serue for those thre denominations, poundes, shillynges, penies: but what if I had, ob. q̄, q, & c? M. You think you bee at Oxforde styll, you bryng forthe so fastе your q and c. These three lines, as I haue sayde, doth serue for thre denominations, such as thei be, as here thei do serue for

poundes

poũdes, shyllynges, & penies: but if you haue no poundes in your summe, than may they serue for shillynges, penies, and halfepenies, yea for q, q, and r, if you haue no greater denomination, so that you remembre that the vpright line serueth for the greatest denomination, and the highest thwarte line for the next, and the lowest for the leaste. And so if you haue foure denominations, you muste make your crosse with so many lynes, And if that your summe be of more denominations make so many lines in your crosse. And thus will I make an ende of Additino.

Examples of addition.

li.	s.	d.
26258	7	6
4163	5	1 2
2812	4	2
47	1	4
33 2	18	4

li.	s.	d.
340	17	10
28	6	8
13	13	4
383	17	10

The proofes.

Another example.

li.	s.	d.	q.
22		6	2
2	3	4	
	10	2	3
24	14	1	1

E.ij.

Scholar.

Then haue I learned the two firste kyndes of Arithmetike, now (as I remembre) doeth folowe Subtraction, whose name me thinketh dooeth sounde contrary to Additiō. M. So is it in dede: for as Additiō increaseth one grosse summe, by bryngynge many in to one, so contrary waies, Subtraction diminissheth a grosse summe by withdrawing of other frō it: so that Subtraction or rebatynge is nothyng els, but an arte to withdrawe and abate one summe from an other, that the remainer mai appere. S. What cal you the remainer? M. You maie perceiue by the name S. So me thinketh, but yet it is good to aske the trouth of all suche thynges, leaste in trustyng to my owne coniecture I be deceaued. M. So is it the surest waie. And as I see cause, I will styl declare thynges vnto you so plainely that you shall not nede to doubt. How be it, if I doe ouer passe it sometimes (as the maner of men is to forgette the smal knowledge of them to whome thei speake) then doe you put me in remembraunce your selfe, and that waie is surest. And as for this worde that you laste asked me, take you this

des-

description: The remayner is a summe left after dewe worckyng which declareth the excesse or difference of the two other numbres: as if I wolde deducte 14 out of 18, there shoulde remaine 4, whiche is called the remainer, & is the differēce betwene those two numbres 14 & 18. S. I perceaue then what subtraction is. Now resteth to know the art to worke by it. M. That shall you do by this meanes: Fyrste you muste consydre, that if you shoulde go about to rebate, you muste haue two sundry summes proposed, the first, whiche is your grosse summe or summe totall, and it muste be set highest, and then the rebatement or summe to be withdrawen, whyche muste be sette vnder the fyrste (whether it bee in one parcell or in many) and that so, that the firste figures be one iuste ouer an other, and so the seconde, and thyrde, and all other folowyng, as you dyd in Addition: then shall you drawe vnder them a lyne, and so are your summes dewely set to begyn your worckyng. Then begynne you at the right hande (as you did in Addtion) and withdraw the nether numbre out of the higher, and if there remaine any thynge, write that right vnder them benethe the lyne, and if there remaine nothing (by reason that the two figures were equall) then write vnder them a cyphre of nought. And so dooe you with all the other figures, euer more abatyng Remainer

eyng the nether out of the hygher, and wryte vnder them the remayner styll, till you come to the ende. And so wyll there appere vnder the lyne what remaynethe of youre grosse summe, after you haue deducted the other summe from it, as in this example: I receaued of your father 48 ß, of whiche I haue layd out for you 36 ß, nowe wolde I knowe what doth remayn, and therfore I sette my numbres thus in order: Fyrste I wryte the greatest summe, and vnder hym the lesser, so that the figures at the right syde be euen one vnder an other. and so the other thus.

ß.
48
36

Than do I rebate 6 out of 8, & there resteth 2, whiche I wryte vnder them right beneth the line thus.

ß.
48
36
2

Than I go to the seconde figures, and doo rebate 3 out of 4, where there remaineth 1, whiche I wryte vnder theym right, and than the holle summe and operation appeereth thus.

ß
48
36
12

Wherby it appereth, that if I withdrawe 36 out of 48, there remaineth 12. S. Nowe will I proue in a greater summe: And I will subtracte 2367924 oute of 3468946. Those summes I set in order thus.

3468946
2367924

Than doo I begyn at the ryght side, and deducte 4 out of 6, & there resteth 2, whiche I wryte vnder them. Than go I to the seconde figures,

ees, and withdraw 2 out of 4, and there remayne 2, whiche I sette vnder theim also: than I take 9 out of 9, and there resteth 0, which I write vnder them, for you say, that if the figures be equall, so that nothyng remayne, I muste write this cypher 0 vnder theim. M. It was well remembred, now go foorth. S. Than come I to the fourth place and draw 7 out of 8, and there remaineth 1 whyche I wryte vnder theim also; than in the .v. place I take 6 from 6, and there resteth nought, for it I write vnder them a cypher 0: than in the .vj. place 3 rebated frō 4, remaineth 1, which I write vnder them: and likewise in the .vij. and laste place 2 taken frome 3 there is left 1, whiche I write vnder them. So haue I doon my holle woorkynge, and my summes appeere thus.

78964
45823
33141

3468946 whereby I se, that if I rebate
2367924 2367924 out of 3468946
1101022 there remaineth 1101022. M.

This is well done, and that you maye be sure to perceaue fully the art of Subtraction, let me se how can you subtract 529847322 out of 8250003456. S. Fyrst I set downe the greattest summe, and than I write vnder it the lesser numbre, begynnyng at the right syde, and than my figures will stande thus.

8250003456
529847322

Than take I 2 from 6, & the rest is 4, which I write

8734

vnder them, then doe I withdrawe 3 from 5, and there remaine 2, whiche I write vnder them: then take I 7 out out of 4, but that I can not, what shall I now doe? M. Marke well what I shall tell you now, how you shall do in this case and in all other like. If any figure of the nether summe be greater then the figure of the summe that is ouer hym, so that it can not be taken out of the figure ouer hym, then muste you put 10 to the ouer figure, and then consider how muche it is: and oute of that hole summe withdrawe the nether figure, and write the reste vnder them. Can you remembre this? S. Yes that I truste I shall. Now then in my example where I shulde haue taken 7 out of 4 and coulde not, I put 10 to that 4, whiche maketh 14, from it I take awaie 7, and there resteth 7 also. whiche I write vnder them. M. So haue you done well, but now muste you marke an other thyng also: that when so euer you do so put 10 to any figure of the ouer numbre, you muste adde one still to the figure or place that foloweth nexte in the nether lyne, as in this example there foloweth 4, to whiche you muste put 1 and make hym 5, & then go on, as I haue taught you. S. Then shall I saie 4 and 1 (whiche I must put to hym for the 10 that I added to 4 before) make 5, which I shuld take out of 3. but that can not be, therfore muste I

put

put to it also 10 and then it will be 13, from whiche I take 5, & there resteth 8 to be writ ten vnder them: and because of that 10 added to the 3, I muste adde 1 to 8 that folowyeth in the nether lyne, and maketh 9, whiche I shulde take out of 0 and can not, therfore I put therto 10 & that maketh 10, from 10 I take 9, & there remaineth 1, whiche I write vnder the. Then do I adde 1 likewise to the nexte figure benethe which is 9, and & maketh 10, that 10 shulde I take out of the figure aboue, but I can not, for it is 0, therfore I put 10 to it, and so take I 10 out of 10, and there resteth 0 to be written vnder them: then come I to the nexte figure whiche is 2, and to hym doe I adde 1, whyche maketh 3, & 3 I can not take out of naught, therfore of that naught I make 10, & thense dooe I take 3, so remaineth there 7 to bee written vnder them: lyke wise doe I take 1 to 5 that folowyeth, & then is it 6, that wolde I take out of 5, and can not therfore I take 10, to that 5, and make it 15, from whiche I rebate 6 and there remaineth 9, whyche I write vnder them: now haue I spent all the nether figures, & what shall I do more? M. you shulde haue added 1 to the nexte figure folowynge (if there had ben any) because you added 10 to the laste figure before of the ouer lyne. But seeynge there is no figure folowynge, you muste adde that 1 to

the place folowynge, and then deducte that 1 from the nūbre aboue. S. Then shal I saie bycause I borowed 10 to the ouer 5, I must put 1 in the nexte place beneth, that is vnder 2, then muste I subtracte that 1 from 2 and there resteth 1 to be writtē vnder that 2 in the ninth place: now I haue no more to subtracte, for there is neuer a figure remaynyng beneth, nother yet any vnitee to be added, because I borowed not 10 to the figure laste before, and yet is there 8 remaynyng in the ouer lyne, whyche (I thynke by reason) shoulde be set at the ende of the figures in the lowest rowe, whiche is vnder the lyne, for because there was nothynge taken from it. M. That is well considered, and reason teacheth so in dede: how be it (as I saied before) I will omitte the reasons tyll another more conuenient tyme, and wyll onely at this time teache you the practise of the arte, & you maie exercise your selfe somwhat in it, in the meane tyme, by proper & witty questions, as I wyl teache you some before we departe. And when you haue well occupied your mynde and quickened your witte in it, then will I afterwarde at your returne, enstructe you in the reasons of all this worckynge, that then you maie worcke perfectely, when you se the reasons in euery thyng, why you shulde so do. S. Syr, wel I maie praie to God to recompence this your

good

goodnes, but surely I can not doe it with any temporall benefyte, neither any other mā in my behalfe: nor what treasure is there to be compared to the riches and treasure of ler vnnge? But syr (I beseche you) shall I alwaies when any numbre so remaineth alone (as this 8 dyd) wrire hym vnder the lyne straight against his owne place? M. Yea what els, whether thei be one or many: and this well remembred, you haue learned subtraction. How be it, because of certeine thinges that might deceaue you, if you dyd not take good hede to your worckyng, I wyl propose to you an other exāple of many nūbres to bee subtracted, as thus. I receaued of a frende of myne to kepe 2869 crownes, of whiche at one tyme I deliuered hym againe 500, at an other tyme 368, and at an other tyme 440, and an other tyme 80, and an other time 64: now woulde I knowe how many doeth reste behynde. Therfore firste I set downe my gros sūme & vnderneth him I set all the parcels, thus, & vnder them a double lyne.

	2869
	500
	368
	440
	80
	64

Then firste I begynne at the fyrste place and gather together the summe of all those lynes (saue the ouermoste) in theyr firste figures, and so do I with all the figures of the seconde place, and so forthe, as I did in addition, saue

saue that I leaue out the highest row of nũbres: and that summe so gathered betwene the lynes, doo I subtract out of the highest rowe of numbres, and the remaynes doo I set vnder the nethermost lyne, as for exãple:

I set the summes as before: than doo I gather the fyrst figures togyther, where I fynde but 4 & 8 that make 12, for thre cyphres encrese no summe in Addition, as as you lerned before, of the 12 therfore doo I write the digite 2, and keepe the article in my mynde tyll I come to the seconde places, where I fynde 6, 8, 4, 6, that make 24, to them I put the article in my mynde, and it is 25, of whiche I write 5 vnder the seconde place, and keepe the digite 2 in my mynde for the iij. place, where I fynde 4, 3, 5, that make 12 to the whiche I adde the 2 in my mynd, and that maketh 14, therof I write the 4, vnder the thyrde place, and because there remayneth no more figures to bee added, I write the digite 1 in the fourth place, as you see in the example. Than come I to subtractyng of this summe, and I shal subtract the summe betwene the lynes, from the ouermost summe, saying: 2 from 9 remayn 7, to be writen vnder them, benethe the lowest lyne,

2	8 6 9
	5 0 0 3
	3 6 8
	4 4 0
	8 0
	6 4
1	4 5 2
1	4 1 7

lyne: than in the second place I take 5 frō 6, and there resteth 1 to bee written vnder them. Than in the thyrde place, 4 from 8, resteth 4. In the.v. place 1 from 2, remaineth 1. And thus I see, that after those 5 summes are subtracted frome 2869, the remainer is 1417. Scho. This I perceaue: but is there no shorter waie & more spedier? Maist. Yes, whan you are a while exercised in it: for you may as fast as you can gather the numbres togither, withdrawe theim out of the highest summe: if so be it, that all the parcelles whiche you doo gather doo not exceede nyne, but and yf they exceede nyne, than muste you subtracte onely the digite that is in it, and reserue the article tyll the nexte place, where you shall adde it with the other figures, and so subtract the whole out of the figure aboue them: but and if in this place the summe of the parcels do exceede 9, then (as I sayd before) subtracte the digite only, and reserue the article to the next place and so stil go forth, tyl you haue ended your worckyng: as for example in the last summes proposed, I gather fyrst in the fyrst place 4 and 8 that maketh 12, of which I deducte the digite 2 oute of 9, and write vnder the remainer, whiche is 7, and the article 1 I kepe in my mynde. Than in the second places I gather the parcels 6, 8, 4, 6, whiche amount to 24, to that I adde the article 1,

whyche

whiche I haue in my mynde, and than is it 25, then doe I take 5 (that is the digette in this numbre) from 6, that is in the seconde place of the highest summe, and there remaineth but 1 to bee written vnder them, and now doe I kepe the article 2 in my mynde styll. Then in the thyrde place, 4, 3, 5, maketh 12, and the article 2 in my mynde maketh 14, then take I 4 (which is the diget) from 8 that is ouer them, and there resteth 4, whiche I write vnder them. Then haue I the article 1 yet in my mynde, whiche I shulde adde to the parcels next folowyng, but seyng there is no numbre folowyng, I take that diget alone, and deducte hym out of the nexte summe aboue, whiche is 2, and then is the remainer 1, which I write in ỹ fourth place vnder 2. Lo now haue you a shorter waie. S. I lyke bothe waies wel, and I perceaue bothe well: yet as in the one, the worckyng semeth somewhat longe, so in the other it leaueth veary muche (me semeth) to remembrance, and therfore maie cause errour quickely, excepte a man haue a quicke & an exercised remembraunce. M. What, wolde you then haue suche a waie, that shulde not be so longe as the one, nor so shorte as the other? S. Yea, if there were any suche. M. Then doe thus: styll as you gather your parcelles when thei excede a diget, and maketh 10 or more, take the article and write hym be-

betwene ii. lynes (as in ỹ firste example) vnder the nexte place towarde the lefte hande, and then deducte the dygette from the figure that is ouer hym, and wryte the remainer. And then when you gather the nexte parcelles, you shall adde to them the figure that is vnder them, betwene the ii. lynes, and if it excede 9, doe as I saied before, wryte the article vnder the next place betwene the lynes, and subtracte the diget from the figure that is ouer those parcels: and if that all the parcelles together, and the numbre betwene the lynes doe make but a digette, then deducte it holy from the figure aboue: as in this example. I wolde subtracte 40308964 out of 40308964, these 20003428 iii. parcelles 20003428 10002342 10002342. 10101461, 10101461 therfore I set them firste in order dewe, and then I gather the parcels of the fyrste place, whiche are 8, 2, 1, that is 11, of whiche I take awaie the article, and set hym vnder the seconde place betwene the lines, & the diget 1 ỹ remaineth I deducte out of 4, and there resteth 3 to be wrytten vnder the fyrste place benethe the lowest lyne. Then come I to the seconde place, and gather the parcels of it 6, 4, 2, & the 1 betwene the lines which make 13, of whiche I take the article and set hym vnder the thyrde place, betwene the lynes, and

and the digite 3 I wrote vnder the seco nd e place benethe the lowest lyne. Than in the thyrde place I fynde 4,3,4, whiche with the 1,, betweene the lynes doue make, 12, therfore I wrote the article agayn vnder the foure the place, and the digite 2, I take from 9, and there remaynethe, 7, whyche I wrote vnder theim benethe the lowest lyne. And thanne come I to the fourthe place, where I gather 1,2,3, and the 1 betwene the lynes, that maketh, 7, whiche bycause it is but a digite, I plucke from 8, and the remainer is 1, and must be written vnder them in the fourthe place. Than come I to the fyfte place, where are onely. vij. cyphres, whiche make nothyng, than shuld I take, that is to saie, nothyng, frome the figure ouer theim, whiche is also a cyphre, therfore I must saie thus: if I take nought from nought, there remaineth nought, so must I wrote a cyphre vnder theim. Than in the. vj. place I finde but 1, whiche I take out of 3 ouer hym, and the remayner is 2, that must be written benethe the lowest lyne in the .vj. place. So go I to the .vij. place, where I fynde onely cyphres, and in the grosse summe ouer them a cyphre also, therfore muste I wrote theyr remayner (whiche is nothyng) with a cyphre also. Than in the. vij. and last place, I gather 1,1,2, that make 4, which if I take out of that 4 that is ouer theim, there wyll nothyng

thyng remayn, and that must be noted with a cypher benethe the lowest lyne, as I haue often said. And thus haue I ended my worke and the figures stande thus.

Scho. Syr, I remembre you taught me that cypheres shuld not come in the last place, forbecause they serue onely to increse the valew of other figures whiche folowe them, and serue not for those figures that go before them: and now in youre exemple you haue set. ij. cypheres in the two last places.

40308964	
20003428	
10002342	
10101461	
111	
00201733	

Mai. I cõmend you for your remembrance, and truthe it is, I shuld not haue sette them here, but onely bycause that I wolde make you playnly to perceaue the art of subtraction, therfore seyng you doo now perceaue it, whan so euer you shuld write down a cypher loke whether any other figures be yet behynd and if not, than let go the cypher also, for it nedeth not to write hym in any later places where no other figure doth folow, except it be (as I did) to teach the vse of subtraction the plainer, therfore my figures must stãd thus, whã I haue ended my woorke. S. So I wold thinke by that you taught me before. And now I thinke I could sub

40308964	
20003428	
10002342	
10101461	
111	
20173	3

tract any like summes. M. So male you, if you haue marked, what I haue taught you. But because this thyng, as all other) muste be learned surely by often practise, I wil propounde here.ii. examples to you, whiche yf you often doo practise, you shall be ripe and perfect to subtract any other summe lightly, for in it is conteyned all the obseruaunces of holle numbre. And because you shal perceaue somewhat both how to doo it, & also whether it be well doon, whan you haue proued to do it, therfore haue I writen vnder them bothe their remainers.

308964	125614
103145	342
102597	681
101024	201
11	11
2198	124390

Scho. Syr I thanke you, but I thynke I myght the better doo it, if you did shewe me the woorkynge of it. M. Yea but you muste proue your selfe to do som thynges that you were neuer taught, or els you shall not be able to doo any more than you were taught, & wer rather to learne by rote (as thei calle it) than by reason. And again there is nothyng in this exãple or any other of hole numbre, but I haue taught you the rules of them all ready. S. Than I trust by practise to atteyn the vse of it. And is this all that I shal lerne

of

of ſubtraction? M. Yea, ſaynge that (as you haue ſene in Addition) there are broken numbres or fractions, in which the worcking is not muche vnlike, yet without ſom inſtructions be geuen of it, it might ſeme to a lerner more difficult than in dede it is, therfore I will brefely ſhew you the vſe of it onely by one exãple or .ij. A certayn man owed to me 14 li. 12 ſ 8 d, of which he paid me at one tyme 4 li, 6 ſ 8 d, at an other tyme 3 li, & at an other tyme 2 li, 3 ſ 4 d, and laſte of all 6 ſ 8 d, now wold I know, what remaineth vnpayde yet: Therfore I ſet my ſummes thus.

	li.	ſ.	d.
	14	12	8
	4	6	8
	3		
	2	3	4
		6	8

S. Syr I prai you, why do you write 2 li? for the cõmon ſpeche vſeth rather to ſai 40 ſ. M. We muſt here vſe the denomination that is greatteſt in any ſumme, ſo that we maie not write, accordyng as we vſe to ſpeake, ſayng 16 d, 18 d, or like wais, 7 grotes, 8 grotes, 24 ſ, 40 ſ, 48 ſ, and ſuche other, but we muſt write euery denomination, ỹ is in any ſumme by it ſelf, namely ſhillynges, & poundes, ſo muſt we write for theſe ſummes now named, 1 ſ 4 d, 1 ſ 6 d, 2 ſ 4 d, 2 ſ 8 d, 1 li, 4 ſ, 2 li. 2 li. 8 ſ and ſo forth of other like. S. So that we may not write in arithmetike penies, whan the ſumme amounteth to ſhyllynges, nor ſhyllynges whan the ſumme maketh poundes.

Nowe it pleaſe you ende your exaumple.

M. Whan my ſummes ar ſo ſette as I ſhewed, than muſte I begynne with the ſmalleſt denomination, ſaiyng, 8, 4, 8, are 20, which ſumme becauſe it is pens, and 12 penies doo make 1 ſ, I muſt take frome that 20, whiche cometh of the iij. parcels 12, & for them wrote 1 betweene the lines vnder the ſhillynges: than the 8 d, that remaineth, I take oute of the hyg heſt ſumme (whiche is 8 alſo) and than remayneth naught, wherefore vnder the penies I wrote nothyng. Than come I to the ſhillynges, & gather the parcels 6, 3, 6, which with the 1 betwene the lines make 16, that muſt I take out of the ſumme that is ouer it: but ſeyng that ſumme is but 12, I can not take 16 out of 12, I muſt borow 1 of the 14 li. & put to the 12, & that maketh 32, for 1 li. is worth 20 ſ: than take I 16 out of 32, and there reſteth 16 to be written vnder the ſhillynges. Than come to the poundes, whoſe parcels are 2, 3, 4, that is in all 9, & 1 more muſt I adde therto, becauſe of the 1 that I borowed before vnto the 12 ſ, and than is there 10, whiche I muſte take oute of 14, ſo dooth there remayne 4 to be written vnder the poundes: ſo dooth my remayner appere

li.	ſ.	d.
14	12	8
4	6	8
3		
2	3	4
	6	8
	1	
4	16	

pere to be 4 li, 16 s. S. This do I perceaue very well, and if there be none other thynge to bee learned in subtraction, than maie I come to Multiplication, for that you reknen next. M. We haue doon in dede with the art of subtraction, as touchyng the woorkyng. But yet before we go to multiplication, I wil instructe you howe to examine your woorke, whether it be well done or no, and that is by castyng awaie 9 as often as you can fynde it (as you dyd in Addition) sauyng that you must here examine the highest nombre alone and note the residue of it at a lyues ende, as you dyd in addition. And whā you haue don with the highest number, than examine all the other togither, castyng thens 9 as often as you can: & if the remainer be like the other than haue you doon wel. But if you haue diuers denominations in your summe, yet for theim all shal you make but one seuerall line as you dyd in Addition, remembryng to begyn the examination at the greatest denomination, & to double the remainer of poundes and triple the remainer of shillynges, as you dyd also in addition. As for a proofe, I will examin this woorke, wherin the highest line I fynd of poundes 14, from thens I bate 9, and there resteth 5, which I do double, bicause thei ar poū-

li.	s.	d.
14	12	8
4	6	8
3		
2	3	4
	6	8
4	16	-

des, and than ar they 10, therto I adde the 12, and it maketh 22, from whiche I take 9 twise, & there resteth 4, whiche bicause they ar shillynges, I triple, & than ar thei 12, ther to I adde the 8, and than ar they 20, thense take I twise 9, and yet resteth 2, whiche I write at the one end of a line thus, 2 ——

Then I examine all the other parcelles, and the remayner togither, euery denomination by it selfe. And fyrst of poundes I fynde 4, 3, 2, 4, that is 13, from whiche I take 9, & there resteth 4, that doo I double, and it maketh 8, to it doo I put the shillynges 6, 3, 6, 16, that is 31, for the 1 betweene the lynes must not be reckned, nor none in that space, and that maketh in all 39, where hens I take 9. iiij. tymes, and there remaineth 3, that doo I take. iij. times & it is 9, wherfore I cast it away: than do I take the penies 8, 4, 8, that make 20, from whiche I take 9 twise, and there resteth 2, whiche I write at the other ende of the profe lyne, and bycause I see that those. ij. numbres ar equall, I saie that I haue well wrought. And if you will, you maye make for euery denomination a line, as you lerned in addition: but then must you begyn your examination at the smallest denomination, as you dyd in addition, for their profe is al togither like, sauyng ỹ in addition you examined the nethermost summe alone, & all the other togither, & in subtractiõ

you

you must examine the highest numbre alone & all the other togither. And if you marke it well, it is euen al one, for that summe that in Addition is lowest, in subtractiō is highest, and that summe is called the grosse or totall summe. Therfore if you marke what I sayd in addition, you may easily perceaue what is to be don for the profe of subtraction: And to thentent ẏ you may perceaue it the better, I will shewe you an other profe of subtraction & that shal be by addition thus. Draw vnder the lowest numbre, which is your remainer, a line, than adde that nūbre, and al the other that you did subtract before togither, & write that that amounteth vnder the lowest line, & if the summe that cometh therof be equall to the highest of the subtraction, than was the subtraction well wrought, els not, as for example, in the last summes, which stode thus.

Grosse or totall summes.

li.	s.	d.
14	12	8
4	6	8
3		
2	3	4
	6	8
1	1	
4	16	

Fyrst I adde 8, 4, 8, that maketh 20, wherof I take 12 awai (bicause thei make one shilling) & write for them 1 vnder the shillings and the 8 that is lefte, I write benethe the lowest line, than adde I the shyllyngs 6, 3, 6, 1, 16, that mke 32, frō which I take 20, and for it I write 1 vnder the poūdes, & the 12 that remaineth, I write vnder the shillynges. Than come I to the poūdes,

addyng them togither, which ar 4, 3, 2, 1, 4 that maketh 14, than do I write 14 vnder the li, and so haue I ended the additiō, and I se that the lowest lyne of numbre and the highest be like, wherfore I knowe ẏ I haue well doone, for my figures appere thus.

li.	s.	d.
14	12	8
4	6	8
3		
2	3	4
	6	8
1	1	
4	16	
14	12	8

And thus now haue I taught you the art of Subtraction, and the meanes to proue whether it bee welle wroughte or not. Now & you remembre, I omitted in teachyng the proofe of Addition one waie, whiche I saide was by Subtraction. S Truthe it is, and than was it deferred, bicause ẏ I had not than learned the feat of subtraction, wherby I shuld haue proued it: but nowe I thanke you, I haue wel lerned the art of subtractiō & the proues of it, bothe by 9 & by addition: And now I wold be glad to know how I may proue Addition by subtraction. M. Than marke you this, whē you haue ended your additiō, take the nūbres al that you did adde to the highest summe, & deduct or subtract them frō the grosse sūme that doth result, & if the remainer be like to the highest nūbre, thē haue you don wel, els not: as for exāple. I take one of the sūmes ẏ I did adde before, which was this.

Than

Then doe I come to the mydle numbre, and subtracte that frō the nether numbre begynnyng at the lefte hande, and firste I saie 0, out of 0, there remaineth 0, that write I vnder an other lyne, Then againe 0 (in the seconde place) from 0, remaineth 0, vnder it I write 0 also. Then in the third place 4 out of 2 will not bee, therfore I adde to that 2, 10, and make it 12, from that I take 4, and there resteth 8. Then saie I farther, 9 in the iiij. place & 1 (whiche I muste adde for the 10 borowed beefore) make 10, that muste I take from 6, and because I can not, I adde to the 6, 10, and then is it 16, from then se I take 10, and there resteth 6 to bee written vnder them. Then in the .v. place, where I fynde nothyng written, I muste set 1 for the 10 laste borowed, and that 1 doe I take from the 1 vnder hym, & so remaineth naught, wherfore I write downe a cipher 0. Now haue I done with the subtraction, & yet in the grosse summe remaineth 1 whyche I must set right in the same place, in the remainer, and so the remainer appereth to be lyke vnto the highe ste summe of the addition, as here appereth, wherfore I saie that the Addicion was well wrought, and thus maie you do in any other summe of one denomination or many. Therfore now will I make

106800	
9400	
116200	
106800	
9400	
116200	
106800	

I make an ende of Subtraction, and will instructe you in Multiplication.

Multiplication.

Multiplication is an operation by .ij. summes producynge the thyrde, whiche so many tymes shall containe the firste, as there are the vnitees in the seconde. And it serueth in the stede of many additions, as for example. If I wold know how many are thirty tymes 48, If I shulde adde 48, thirty tymes it wolde be a longe worke: therefore was this worke of Multiplication deuised, which shall do that at ones, that Addition shulde dooe at many tymes. S. I perceaue the commoditee of it partely, but I shall not see the full profyt of it, tyll I knowe the hole vse of it. Therfore syr, I beseke you teache me the worckyng of it. M. So I iudge it best, but because that great summes can not be multiplied, but by the multiplication of digettes therefore I thynke beste, to shewe you fyrste the arte of multiplyng them, as when I saie, 8 tymes 8, or 8 times 9. &c. And as for the small digetes vnder 5 it were but foly to teache any rule, seyng thei are so easy ꝑ euery childe can

Multiplication of digetes.

doe

doe it: but for the multiplicacion of the greater digetes thus shall you do. First set your digettes one ouer the other right, then loke how many eche of them lacketh of 10, and write that against eche of them, and that is called the differences: as if I wolde knowe how many are 7 tymes, 8, I muste write those digetes thus. The deffe-rences.

8
7

Then doe I loke how muche 8 doth differ from 10, and I fynde it to be 2, that 2 doe I write at the right hande of 8, thus.

8 2
7

Then doe I take the difference of 7, lykewaies from 10, that is 3, and I write that at the right side of 7, as you se in this example.

8 2
7 3

Then doe I drawe a lyne vnder them, as in Addition, thus.

Then do I multiply the two differences saiyng, 2 tymes 3 make 6, that muste I euer set vnder the differences benethe the line: then muste I take the one of the differences (whi che I wil, for all is lyke) from the other diget: not from his own, and that that is lefte, muste I write vnder the digettes, as in this example.

8 2
7 3
5 6

If I take 2 from 7, or 3 from 8, there remaineth 5, that 5 must I write vnder the digetes, & then there appereth the multiplication of 7 tymes 8, to be 56, & so like of any other digetes, if thei be aboue 5, for if they bee

be vnder 5, then wil their differences be greter then them selfe, so that they can not be taken oute of theym, and againe suche litle summes euery chylde can multiply, as to saie, 2 tymes 3, or 4 tymes 5, and suche lyke. S. Truthe it is, and seyng me semeth, that I vnderstande the multiplying of the greater digettes, I will proue by an example how I can doe it: I wolde knowe howe many are 9 tymes 6. M. It is all one in valewe to saie 9 times 6, or 6 times 9, but yet the order is beste to put the lesser summe first saiyng, 6 tymes 9, and so of all other summes. S. then wolde I know how many are 6 tymes 9, therfore I set the digettes thus.

9
6

Then do I set their differēces at their right side, the difference of 9 whiche is one againste it, and the difference of 6 whiche is 4 againste it also, as in this example.

9 1
6 4

And vnder them I drawe a lyne, then do I multiply the digettes together, saiyng 4 time 4 maketh 4, that 4 do I write vnder the differēces thus. Then take I one of the differences from the other deget, as 1 from 6, or els 4 from 9, & eche waies there resteth 5, whyche I doe wryte vnder the digettes, and so appereth the multiplicacion of 6 tymes 9 to be 54. Thus I see the feate of this maner of multiplicacio of digets. M

9 1
6 4
5 4

Now

Now mought you go straight to the multiplication of greater numbres, saue that both for your ease and sureree in worrkyng, I will drawe you here a table, wherby shall appere the multiplication of all digettes, and this is it.

A table to multiply all digettes by.

1	1	2	3	4	5	6	7	8	9
2	4	6	8	10	12	14	16	18	
3	9	12	15	18	21	24	27		
4	16	20	24	28	32	36			
5	25	30	35	40	45				
6	36	42	48	54					
7	49	56	63						
8	64	72							
9	81								

In whyche figure whan you woulde knowe any multiplication of digites, seke your first or last digite in the blacke squares, and from it go ryght foorthe towarde the right hand, tyll you come vnder the digitte of youre seconde digite, whiche is in the hyghest rowe, and than the numbre that is in the meynge of their bothe squares, is the multiplication that amounteth of theim. As if you woulde knowe by this table the multiplication of 7 tymes 9, seke fyrst 7 in the blacke squares, and than go ryght foorth towarde the ryght

hande,

hande, till you come vnder 9 of the highest rowe, & in the metyng of their squares, you may se 63, which is their multiplicacion. S. This is veary good and ready, and so maie I fynde the multiplicacion of any digettes.

To multipli greater summes.

But now how shall I doe in greater summes

M. When you wolde multiply any summe by an other, you shall marke, that it is the meteste order to set the greateste numbre highest, whiche is the place of the numbre that muste be multiplied: and lyke waies the lesser numbre vnder it, for that is the place of the multiplier or multiplicatour, that is to

Multiplier.

saie, the numbre, by which multiplication is made, and is in englyshe al waies put before this worde, tymes, in suche speakyng, when

Tymes.

I saie, 20 tymes 70, and the numbre that foloweth this worde, tymes, is that whiche muste be multiplied. Therfore when I wold multiply one numbre by an other, I muste write the greateste highest, and the lesser vnder it, as in addition. And vnder them must I drawe a line, as for example. If I woulde multiply 29 I muste sette them thus.

264
264
29

Thā must I multiply euery figure of the higher rowe, by euery figure of the nether rowe, and that that amounteth I must set vnder the lyne, as thus: first I do multiply 4 by 9 saiyng, 9 tymes 4 (or 4 tymes 9 which is all one) & that maketh 36, as the table

table before of digettes doeth declare, of that 36 I muste write the 6 that is the diget vnder the 9, and the 3 in the next place toward the lefte hande.

Then come I to the seconde figure of the higher rowe & saie, 9 tymes 6, make 54, of wh'che I write the 4 vnder the 3, and the 5 vnder the nexte place (as reason willeth me) thus.

Then come I to ỹ next figure, whiche is 2, and dooe multiply it by 9, and that maketh 18, wherof I write 8 vnder the thyrde place, and the article 1, in the fourth place, thus.

And then haue I ended the firste figure of the multiplier. Then begyn I with the nexte figure, and multiply it into all the higher figures, as thus.

Fyrste 2 tymes 4, make 8, that do I write vnder the seconde place (for euer more the diget, or fyrst figure of the multiplication that amounteth of the fyrste figure of the higher numbre, muste be set vnder the multiplier of it, and the other in their order toward the lefte hande. S. I vnderstand you thus, that the diget of the summe amountynge of the multiplication of ỹ firste figure of the higher rowe, by the firste figure of the lower rowe or multiplier, muste be set vnder the first place:

264	
9	
36	
264	
9	
536	
4	
264	
9	
1536	
84	
8	

and

and that that amounteth of the same firste figure by the seconde multiplier, muste bee set vnder the seconde place: and so of the other, if there be more multipliers. M. So meane I in dede: and if there amount but a dygette, than muste it be set vnder the multiplier. And now to go forthe, I multiply by the same 2, the seconde figure of the higher towe, whiche is 6, saiyng 2 tymes 6, make 12, wherof I write ẏ diget 2 vnder the thirde place, and the article 1 I write vnder the fourth place.

Then doe I multiply the laste figure of the higher sume, by that same 2, saiyng, 2 tymes 2 is 4, whythe I wryte vnder the fourth place, and so haue I ended the hole multiplicacion, and the summes stande thus.

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 264
  29
----
1536
 184
 28
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Than muste I drawe a lyne vnder al those summes that amount of the multiplicacion, and muste adde all them into one summe, as in this exaumple you maie see, where in the firste place I fynde but 6, and therfore write I it vnder the lyne: then in the seconde place 8, 4, 3, make 15 wherof I write 5, and kepe 1 in my mynde and so forthe as you lerned in Addicion & so appeareth the hole

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 264
  29
----
1536
 184
 428
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 264
  29
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1536
 184
 428
----
7656
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summe

summe to be 7656, whiche amounteth of the multiplication of 264 by 29. Sco. If ther be no more to be obserued in it, then can I doo it I suppose, as by this exaumple I shall proue: I wold multiplie 1365 by 236, wherfore I set theim thus.

1365
236

Than doo I multiply 5 by 6, saiyng: 6 tymes 5 make 30, of whiche I write the cyphre in the first place, and the article 3 in the second place.

1365
236
30

Than doo I by the same 6, multiply the second figure of the higher summe, which is 6, saiyng: 6 tymes 6 make 36, of whiche I write the 6 vnder the second place, and the 3 vnder the thyrd place.

Than doo I multiplie the thyrde figure, whiche is 3 by the same 6, and that maketh 18, of that I set the 8 vnder the third place, and the 1 in the fourth place.

1365
236
3330
6

Than come I to the laste figure of the higher summe, and multiplie it by 6, saiyng: 6 tymes 1 make 6, that doo I write vnder the fourth place.

1365
236
13330
86

And so haue I eanded the fyrste multiplier: than begyn I with the seconde multiplier, and saye fyrst 3 tymes 5, that maketh 15,

1365
236
13330
6866

of which I set the 5 vnder the secõd place, bicause that the multiplier is there, & the 1 I

set vnder the thyrde place. Than come I to the secõd figure that is 6, and multiplie it by 3, whyche maketh 18, of which I set the 8 vnder the thyrd place, and the article 1 in the fourthe place.

Than come I to the thyrd figure which is 3, and multiplie it by 3, saying: 3 tymes 3 make 9, which bicause it is but one digitte, I set vnder the fourth place.

And then comynge to the last figure 1, I multiplie it by 3, and it maketh 3, which I set in the fifte place, and than haue I ended.ij. of the multipliers, & the summes stande thus.

Then come I to the thyrd multiplier, & multiplie it into euery figure of the higher sum, and first I say: 2 times 5 make 10, of whyche I sette the cypher vnder the multiplier in the thyrd place, and the article 1 in the fourth place.

And so multiplyng the second figure 6 by that same 2, there amounteth 12, wherof I write the digite 2 vnder the fourth place, & ỹ article 1 vnder ỹ.v.place. Now do I multiplie by the same 2 the third figure of the higher sume, whyche

1	3	6	5	
	2	3	6	
1	3	3	0	
6	8	6		
1	1	5		
3	9	8		
1	0			
8				

1	3	6	5	
	2	3	6	
1	3	3	0	
6	8	6		
1	1	5		
3	9	8		
1	0			
2				

1	3	6	5	
	2	3	6	
1	3	3	0	
6	8	6		
1	1	5		
3	9	8		
1	0			
2				

1	3	6	5	
	2	3	6	
1	3	3	0	
6	8	6		
1	1	5		
3	9	8		
1	0			
2				

1	3	6	5	
	2	3	6	
1	3	3	0	
6	8	6		
1	1	5		
3	9	8		
1	0			
2				

whiche is 3, and that maketh 6, whiche I set vnder the fift place. Than come I to the laste place, and multiplie that 1 by 2, and there amounteth 2, whiche I sette in the sixte place, and then doothe the summes stande thus.

1365
236
1330
686
115
398
110
262

And so haue I ended the whole multiplication. But now (as you taught me) to knowe what this hole summe is, I muste adde all those parcels togyther, and than vnder the lyne wille appeere the grosse or totall summe, that is 322140. M. That is wel done.

S. Than (me thinketh) I wolde calle it well doone, when I knewe whether I had well doone or no. Maist. It maie bee tried by 9 as Addition was, but the surest proue is by Diuision, and therfore I will reserue that, tyl you haue lerned the art of Diuision. And before we passe from Multiplication, I will yet shew you an other manier of multiplication, which ar counted of some men bothe more ready, and more certayn, of which the one differeth nothyng from this that I haue taught you, saue yt it doeth vnderstande all waies the articles, and ioyne theym to the nexte summe, and therefore I wylle declare it onely by an exaumple: If I wouldе multiplye 1542, by 365, I muste sette theym as I sayde beefore,

1365
236
1330
686
115
398
110
262

and then doo I multiplie 2 by 5, and it maketh 10, of whiche I write the [illegible] vnder the fyrst place, and keepe the [illegible] 1 in my mynde.

short article

Than saie I foorth: 5 tymes 4 doo make 20, and the one in my mynde are 21, thereof I write the 1 vnder the second place, and kepe the 2 in my mynd.

Than come I to the third figure 5, saiyng: 5 tymes 5 make 25, and the 2 in my mynd make 27, wherof I write the 7 vnder the third place and keepe the article 2 in my mynde.

Than comyng to the last figure I saie: 5 tymes 1 make 5, and 2 in my mynde make 7, that doo I wryte vnder the fourthe place.

And than haue I ended my fyrste multiplier. Than do I like waies wt the second multiplier, saiyng, 6 tymes 2 make 12, therof I write the digite 2 vnder the seconde place, and kepe the article 1 in my mynde.

Than saie I forth: 6 tymes 4 maketh 24, & the 1 in my mynde make 25, so I set that 5 vnder the thyrde place, and kepe the 2 in my mynde.

Than multiplye I foorthe, saiyng: 6 times 5 maketh 30, and 2 in mynde make 32, wherof I wryte the 2 vnder the fourthe

1542
365
10

1542
365
710

1542
365
7710

1542
365
7710
2

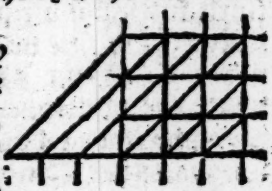
1542
365
7710
52

place, & kepe the 3 in my mynde. Than doo I multiply the last figure 1 by 6, and it maketh 6, to that I adde the 3 in my mind and it maketh 9, whiche I write in the fyfth place. And so haue I ended. if. figures of the multiplier. Than with the third and last multiplier do I like waies, and saie first 3 tymes 2 make 6, whiche I write in the thirde place vnder the multiplier. Than by that 3 doo I multiplie like waies the second figure 4, & it maketh 12, wherof I write the digite 2 vnder the fourth place, & the article 1 I kepe in mynd. Than come I to the third figure 5, sayinge: 3 tymes 5, maketh 15, and the 1 in my mynd make 16, therof I write the 6 vnder the fyft place, and kepe the article 1 in my mynde. Than come I to the laste figure whiche is 1, and multiplye it by 3, and it maketh 3, thereto I adde the 1 in my mynd and it maketh 4, whiche I write in the firt place. And than haue I ended the multiplication, and the figures stande in order thus:

1	5	4	2	
		3	6	5
7	7	1	0	
	2	5	2	
	1	5	4	2
		3	6	5
7	7	1	0	
9	2	5	2	
	1	5	4	2
		3	6	5
7	7	1	0	
9	2	5	2	
	2	6		
	1	5	4	2
		3	6	5
7	7	1	0	
9	2	5	2	
	6	2	6	

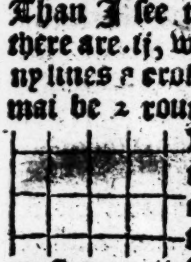
1542
365
7710
9252
4626

whyche parcelles yf I adde into one ſumme, it wyll bee 562830, whiche is the groſſe or totall ſum of all that multiplication. Scho. Wel, this maner of multiplication I perceaue: but what other ſortes haue you? M. There is one waie that is wrought by a checker table made thus.



And looke howe many places your ſum hathe that you wold multipli, ſo many ſquares muſte you make in your table, from the right ſide to the leſte, and ſo many from the higher part to the lower as there be places in your multiplier. Than ſet down your greateſt ſumme fyrſt on the toppe of the table, euery figure in dew order in a ſquare alone, I mean in thoſe ſquares, that be open and vncroſſed. And likewaies in thoſe lyke ſquares, at the right hand, ſette downe your multiplicatour or multiplier, the laſte figure in the higheſt place, and ſo downward, that the fyrſt figure mate bee in the loweſt place. Scholar. Syr, if it pleaſe you, me thynketh than I vnderſtande you beſt, whan you doo not ſtand longe in tellyng the rule before examples, but propoſe ſome example, and then in declaryng it, bryng in the rules withall. M. In dede that wai is eaſieſt for a yong lernar, therfore wyl I euen ſo do. Take this example

ample now: I wold multply 2036 by 23. Fyrst I consider that my greatest numbre hath .iiij. figures or places, and therefore I make so many roumes betwene lines, thus. Than I see that of my .multipliers there are .ij, wherfore I draw also many lines a crosse the other, that there mai be 2 roumes betwene thē, thus.

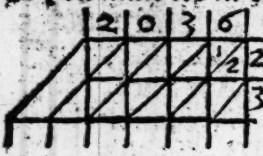


But you must not forget to let the endes of the lynes runne out (as it appereth in this patron) for in those open squares must your .ij. first numbres, and all the total summe be set. Than draw a crosse barre thorough euery close square, so that it may reach downe to the lowest ouerthwarte line, as in this fourme.

And than is youre checkerforme prepared: thā set downe your fyrste or greatest summe on the toppe, and your multiplier on ȳ right side in ȳ opē squares thus.

Than begyn to multiply the firste figure of the higher sum by the highest of ȳ multiplier, saying: 2 tymes 6 make 12, that 12 must you write in the square that is agaynst the 2 and the 6, but of suche maner that the digitte bee sette in the nether corner of the square, and the article in the hygher corner.

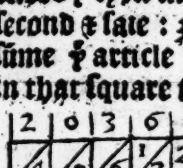
as you mate see in this example.



And so of euery other multiplication, what euer amoũteth you muſt wryte in the commen ſquare, which is againſt bothe thoſe figures by whiche you doo multiplye, and if that ſumme doo make but one digite, than muſt it be ſette in the lower corner of the ſquare: but if it make an article, than wrıte the article in the hygher corner, and let the cypher go (if you wyl) euermore, for here it ſerueth for nothyng, ſeyng the lynes doo diſtincte the places, but if the ſumme amountyng of ſuch multiplication do make a mixt numbre, then write the article in the higher corner, and the digite in the lower corner, as I dydde by that 12: then whan you haue multiplied and ended the firſte figure, come to the next, and multiplie it in like maner, as in ſaiyng: 2 tymes 3 is 6, that 6, bicauſe it is but a digette, you ſhall ſette in the nether corner of the ſquare, next vnder 3, as here foloweth.

Then go forth ſaiyng: 2 tymes 0 is 0, ſet 0 vnder the barre (if you liſt) in the third ſquare. Than forth, & ſay 2 tymes 2 make 4, & ſet in the laſt ſquare vnder the barre, ſo haue you

ended

ended ỹ fyrst multiplier: come therfore to the
second & saie: 3 times 6 make 18, of whiche
sume ỹ article 1 must be set aboue the barre,
in that square that is next to the 3, (as you se
here) & the 3 vnder the barre

Then saie: 3 tymes 3 ma-
keth 9, sette it in the nexte
square beneth the barre, the
3 times 0 is 0, write it in the
nexte squre or let it go, for all is one. S. I
perceaue it wel, for here the lines distinct the
places, wherfore ciphers doe onely serue, &
therfore here thei nede not to be. M. Then
saie farther, 3 times 2 make 6, write that in
the laste square, then will the hole figure stād
thus as here followeth.

S. Now could I me semeth,
doe lyke againe: But howe
shall I do now to gether the
summe? M. Marke fyrste
the order of the places in this figure, and so
shall you perceaue the reason of gatherynge
them into a summe. The slope barres dooe
parte ỹ places, so ỹ the fyrste place is the lo-
weste corner (in all suche figures) of the ne-
thermoste square, nexte the right hande, and
all the halfe squares betwene that barre and
the next, standeth for the seconde place, and
so the roume betwene that & the nexte barre
is the thyrde place, and so forthe. Nowe if
you perceaue this, then muste you adde all

the figures of one place together, as if you had an addition of diuers summes. S. If I vnderstande you right, then must I take here in this example 8 to be in the fyrst place 9, 1, & 2, in the second. 0, 6, 1, in the thirde 6, 0, in the fourthe, 4 in the fifte, and the syxte place hath no figure. M. you saie wel, and the reason is bycause the multiplicacion seruyng to that square, made but a digette. S. Then it is all one, as if thei stode thus. M. euen so it is, and now adde this summe, and there will appere the totall of the multiplication to be 46828. And if you wil se the agrement of this maner of multiplicacion, and the other that you lerned before, then multiply those two summes (that is 2036 by 23) after the first maner without squares S, you mene to set thē thus in order. And then multiply 3 into 6, make 18, 3 tymes 3 make 9, 3 tymes 0 is 0, then thre tymes 2 make 6 whiche muste be set thus. Then doe I lyke waies with the seconde multiplier, saiynge: 2 tymes 6 make 12, 2 tymes 3 make 6, 2 tymes 0 is 0, and 2 tymes 2 make 4, whiche when I adde to the other, then will the hole multiplication stand thus. M. So that you maie see in euery place the same figures here, as thei were in the multi-

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   12
  061
46098
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2036
  23
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2036
  23
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  18
609
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plicaci

plication by squares, though thei differ in height and lownes of places, but beyng added together thei make one summe. And thus nowe ye haue lerned iij. sortes of multiplication, which you lyketh beste, that may you vse. Yet ar there other formes, but sith thei nothing differ from these three in effecte, but onely in settyng of the numbres, I will ouerpasse them til a more meter place and tyme, And now will I enstructe you in Diuision, so that you thinke your selfe sufficiently to perceaue what I haue taught you S. Yes syr I thanke you, but I doe not perceaue how to examine my worke, to trie whether I haue well done or no. M. That is cõmenly vsed by the profe of 9, as you learned before in Addition and Subtraction, saue that it hath this waies diuers from them, first you must make a cros after this maner. Then must you examine your sũme that shoulde be multiplied, and loke what remaineth after castyng awaie of 9, that set you at the one syde of the crosse, then examine the multiplier, and what so euer remayneth in it after castynge awaye 9, as often as you can, wryte that at the other syde of the crosse: then muste you multiply those. ij numbres togeather, and loke what amounteth thereof, if it bee vnder 9, write it at the higher parte of the crosse: but if it be

2036
23
18
609
112
406

be aboue 9, then take thence 9 as often as you can, and write the reste at the head of the crosse, as in the laste example of multiplication, the numbre to be multiplied is 2036, wherin is ones 9, and 2 remaineth, whych I write at one syde of a crosse, thus. Then do I examine the multiplier whiche is 23, wherein there is no 9 but 5 in all, that 5 therfore I sette at the other syde of the crosse, thus. Then doe I multiply 5 by 2, and it maketh 10, from whiche I withdrawe 9, and there resteth 1, that 1 doe I set at the head of the crosse, then doe I examine the grosse summe amountynge of the multiplicacion, which is 46828, wherin I fynde 9 thre tymes, and 1 remainyng, that 1 I set at the fote of the crosse, and then I se it to agree with the other 1 at ỹ toppe of the crosse, and so know I that I haue done well: for if thei two did differ, thē were my worke vaine, and the multiplicacion false. This is the cōmen proue, but the moste certaine proue is by Diuision, of whiche I will anone enstructe you. S. Syr what is the chief vse of Multiplication? M. The vse of it is greater then you can yet vnderstand, how be it these plain commoditees it hath, that if you wolde resolue any great and hole valeure into many smal & lesse portions: as if you wold change poundes into shillynges, pennies or any o-

ther

ther greater or smaller parcels, by multiplication you shall do it spedily and easily. Also if you shulde nede to adde one summe to it selfe, or to any other often tymes, you shall doe it by multiplication muche more spedily, reddily, easily, and surely, then by often and sundry Additions: Take you these commodities grossely shewed for an answere at this tyme, and here after I will more abundantly make you to perceaue the vse of it.

DIVISION.

Scholar.

Ell sys, then in Division I praie you to instructe me. But me thinketh by the name of it, that it shulde be all one with Multiplication: for I call that Division, when any thynge is parted into diuers and many partes. M. You take it as it is taken commenly, how be it, if you marke well, you shall perceaue that it is quite contrary to Multiplication, and dooeth not parte one thynge or few thynges into many, but contrary waies it bringeth many parcels into fewe, but yet so, that these fewe taken toge-

there

ther are equall in valure to the other many, for by Diuision pennies are turned into shyllynges, and shyllynges into poundes: as for example of 120. ℔. it maketh 6. l. so are 12,0 turned into 6 which is a smaller numbre, but then if you consydre the denominatours, you shal se that thei are such, that one of the later is equall to 20 of the firste, and so in valewe the summes are one, though in numbre thei doe farre dyffer, and the later summe is ℣ lesser, & so is it alwaies in Diuision, how be it yet in the worknyng, the same is parted by an other, & therof doeth it take the name, Scholar. I thynke I shall better understande the reason of the name, when I knowe the vse of the worke, therfore now wouldе I gladly learne that. Mayster. Diuision is a partition of a greater summe by a lesser, (therefore as you maie perceaue) vnto Diuision are required ij. numbres, the firste, whiche shoulde bee diuided, and that muste be the greater, and the second, by whiche the other muste be diuided, and that is the lesser, and is called the diuisor. The firste muste be first written, & the second so set vnder it, that the laste figure of the lower numbre be right vnder the last of the highest, contrary wates to the worke of the other kindes of Arithmetike, for in them the two firste figures wer set euer mete one vnder the other but in Diuision the laste figures muste be set

mete

mete, except it chance so that the laste figure of the Diuisor be greater then the laste of the hygher numbre, for then you shall sette the laste of the Diuisour, vnder the last saue one of the higher numbre, as for example: if you shulde diuide 365 (whiche are the summe of the daies of a yere) by 28 (whiche are the daies of a commen moneth) then shulde you set them thus. 365

But if you wolude diuide those 365 28.
daies, by 52 (which is the numbre of weekes
in one yere) then shulde you set them thus:
Lykewaies if I wold diuide the same 365
365 by 4, which is the summe of the 52
quarters of a yere, then must I set the thus.
S. Syr this doe I vnderstande, but 365
how now shuld I do to diuide ỹ one 4
by ỹ other? M. You must begyn with ỹ last figure next ỹ left hande, & se how many times ỹ last figure of the diuisor may be taken out of the last figure of ỹ ouer numbre, & that shall you note within a croked lyne towarde your right hande, as for example: I wold diuide 365 by 28, then set I those two sumes, thus

365 And I loke how many tymes I may
28 tynde 2 (whiche is the laste figure of
the diuisour) in 3 (whych is the laste
of the numbre to bee diuided) and consyderynge that I can take 2 out of 3 but ones, I make a croked lyne at the right hande of the numbres, and within it I sette 1, & that is

called

Quotient numbre.

called the quotient numbre: then bycause that when 2 is taken out of 3 there remaineth 1, I muste write that 1 ouer 3, and deface or cancel the 3 and the 2, then will the figures stande thus. 1

Then muste I go to the next figure of the diuisor, & take it like waies so many tymes out of the figures ꝑ be ouer it, & loke what doeth remain, that I must write ouer thē, & cancel thē, as in this exāple: I take ones 8 out of 16, & there remaineth 8, whiche I must set ouer the 6, and cancell or crosse out the 16 and the 8 of the diuisour: And then will the figures stande thus. And so haue I ones wrought. S. So I perceaue that you take the nether figure not onely out of the other that is right ouer ꝑ, but out of that with ꝑ other also ꝑ remaineth before, and are written toward the lefte hand.

M. So muste you doe, for you muste so take the diuisor out of the ouer numbre, that ther remaine not ouer it so great a summe as it selfe is: for then were your worke in vaine. But yet againe here muste you marke, that when you seke how many tymes the laste figure of the diuisour maie be founde in the numbre ouer hym, that you looke also whether you maie as often fynde all the figures folowing in those that are aboue them, if not take

take your quotient lesse by one, & than proue agayne, and so stylle, tyll you fynd a mete quotient. Whan you haue thus wroughte ones, than must you begyn agayn, and write your diuisour a new, nerer toward the right hande by one place, as in this example: you shall sette 2 vnder 8, and 8 vnder 5, thus.

~~4~~	8		
~~3~~	~~6~~	5	(1
~~2~~	~~8~~	8	
	2		

Thē (as before) seke how many tymes you maye take the last diuisour out of the numbre ouer him. S. That maie I do here 4 times. M. Truth it is, that you maie fynd 2 foure times in 8: but than marke whether you can fynde the figure folowyng so many tymes in the other that is ouer him. Can you finde 8 foure tymes in 5? S. No, nother yet ones. Maister. Therfore take 2 out of 8 ones lesse. Scho. That is 3 tymes. Mayster. Well, then 3 tymes 2 make 6: yf I take 6 oute of 8, there remayneth 2, which 2 with the 5 folowyng make 25, in whiche summe I maie fynde 8 thre tymes also, and therfore I take 3 as a true quotient, and write it within the croked line of the quotiēt before the 1: thus.

~~4~~	8		
~~3~~	~~6~~	5	
~~2~~	~~8~~	8	(13
	2		

Than saie I: 3 tymes 2 make 6, than 6 out of 8 resteth 2, therefore I cancell the 2 and the 8, and wryte ouer it the 2, that dooeth remayne thus.

2
x 8
3 6 4
x 8 8 (13
x

Then doo I take 8 as many tymes out of 25, saying: 3 tymes 8 make 24, and if I take 24 oute of 25, there remayneth 1, so then I cancel 25 and 8, & ouer the 5 I set 1, thus.

x
x 8 1
3 6 4
x 8 8 (13
x

And now haue I don with diuidyng, for I can finde my diuisor 28 no more in the ouer summe. S. No, except you wolde part the 1 that remaineth into 28 partes. M. That is well sayd, and so must we doo in suche cases when there remaineth any thyng: but I wil lette that passe now, and will make you perfecte in holle diuision, and will hereafter teache you peculiarly of broken numbre called fractions. And nowe if you do perceaue the order of diuision, than doo you diuyde this summe 136280 by 452. S. Fyrst I set down the numbre that shuld be diuided, than doo I set the diuisor vnder it, so that the last figure of it, be right vnder the last figure of the ouer nũbre. Thã will it be thus. 136280

M. Can you take the last of 452

your diuisor (whiche is 4) out of 1, whiche is the last of the ouer numbre? S. I had forgotten, because the last of the diuisor can not be taken out of the last of the ouer numbre, in as muche as it is the greatter, therefore must I sette the diuisour one place more forwarde

warde toward the right hãde thus. And then must I looke howe often I may finde the last figure of the diuisor (that is 4) in 13, which thing I may doo 3 tymes, therfore doo I say: 3 tymes 4 is 12, whiche I take out of 13, and there remaineth 1, then do I make at the right hãde of my summes a croked lyne, and write before it my quotient 3, and I cancell 13 and 4, and ouer the 3 I sette the 1 that remayneth, and than the figures stande thus.

1 3 6 2 8 0 (3
4 5 2

Than doo I multiply the same quotient into euery figure of the diuisor, and withdrawe the summe that amounteth oute of the numbres ouer theim: as first I saie: 3 tymes 5 make 15, which I take from 16, and there resteth 1, I cancell therefore 16 and 5, and write ouer the 6 that 1 that remayneth thus.

x x 1
x 3 6 2 8 0 (3
4 5 2

Than doo I saye lyke wais: 3 tymes 2 make 6, which I take out of 12, and there resteth 6, therfore I cancell the 12 and the 2, and ouer the 2 I write 6, that remaineth, thus.

x x 6
x 3 5 x 8 0 (3
4 5 x

Thẽ shuld I set forward the diuisor into the next place towarde the right hãd thus. M. But you mai se, that ouer the 4 is no figure, therfore must you set the diuisor yet forwarder

x x 6
x 3 6 x 8 0 (3
4 5 x
4 5 2

by an other place. And marke whā so euer it chanceth so, that you shuld set forward the diuisor, & that it can not stande there, bicause there is no numbre ouer the laste place, or if there be any, it is lesser than the last figure of the diuisor, thā must you remoue the diuisor yet ones agayn: and bycause ẏ his first place of remouyng fayled hym, therfore shall you write in the quotient a cyphre 0, and if you shuld by chaunce nede to doo so often tymes, for euery time write a cyphre in the quotient The reason of this will I shewe you hereafter. S. Then muste I set my summes thus.

```
   1  1  6
1  3  6  2  8 0   (30
   4  5  2  5 2
      4
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And bicause I remoued the diuisour, so ẏ I ouerskypped one place, I must write a cyphre in the quotient: and than must I seke a newe quotient, as in this example, I must saie: How many times 4 is there in 6, and sithe it can be but ones, therfore doo I write 1 in the quotient, and than saie I, 1 tymes 4 taken out of 6, remaineth 2, I cancelle the 6 and the 4, & write 2 ouer them. Then sai I again: ones 5 out of 28 remaineth 23, I let the 2 stand as it dyd, and ouer the 8 I set 3, cancellyng the 8 & the 5 vnder it, thus

```
      2
   1  1  6  3
1  3  6  2  8 0   (301
   4  5  2  5 2
      4
```

M. You mighte as wel haue said, ones 5 out of 8, & so remaineth 3, but nowe go foorth.

S. That ones 2 out of 0, can not be, what shall I now doo? Mai. Borowe of the nexte numbre that is behynde (for there is 230) & doo as you learned in Subtraction in a lyke case. S. Than must I borow 1 of the 3 comyng behynde next, and make that 0 to be 10, and then take I 2 oute of 10, and there resteth 8, and bycause I borowed 1 of the 3, I muste cancell the 3, and wrote 2 ouer it, than doothe the figure stande thus.

2 2
x x 6 3 8
x x 6 x 8 0 (301
x x x x
x

M. Nowe haue you doone, and yet remaineth 228, and youre quotient dooth shewe you, that yf you diuide 136280 by 452, you shall fynd your diuisor in your greater numbre 301, that is. CCC. tymes and ones, and 228 remaynyng. And in the other exaumple where I diuided 365 by 28, the quotient was 13, and 1 remained, wherby I know that in a yere (whiche conteyneth 365 daies) there are 13 monethes, reckenyng 28 daies (or 4 wekes) iuste to a moneth, and 1 daie more. S. Why than doo we call a yere but 12 monethes? M. Of that at a more conuenient tyme will I fully instruct you, but now it is not cõuenient to entangle your mynd with other thyngs thã do directly pertayne to your matter, therfore yf you can remembre what you haue herde, you haue lerned a short maner of diuision, which

by an other place. And marke whā so euer it chanceth so, that you shuld set forward the diuisor, & that it can not stande there, bicause there is no numbre ouer the laste place, or if there be any, it is lesser than the last figure of the diuisor, thā must you remoue the diuisor yet ones agayn: and bycause ẏ his first place of remouyng fayled hym, therfore shall you write in the quotient a cyphre 0, and if you shuld by chaunce nede to doo so often tymes, for euery time write a cyphre in the quotient The reason of this will I shewe you hereafter. S. Then muste I set my summes thus.

```
   1 1 6
1  3 6 2 8 0   (30
   4 5 2 5 2
       4
```

And bicause I remoued the diuisour, so ẏ I ouerskypped one place, I must write a cyphre in the quotient: and than must I seke a newe quotient, as in this example, I must saie: How many times 4 is there in 6, and sithe it can be but ones, therfore doo I write 1 in the quotient, and than saie I, 1 tymes 4 taken out of 6, remaineth 2, I cancelle the 6 and the 4, & write 2 ouer them. Then sai I again: ones 5 out of 28 remaineth 23, I let the 2 stand as it dyd, and ouer the 8 I set 3, cancellyng the 8 & the 5 vnder it, thus

```
       2
   1 1 6 3
1  3 6 2 8 0   (301
   4 5 2 5 2
       4
```

M. You mighte as wel haue said, ones 5 out of 8, & so remaineth 3, but nowe go forth.

S. Thay ones 2 out of 0, can not be, what shall I now doo? Mai. Borowe of the nexte numbre that is behynde (for there is 230) & doo as you learned in Subtraction in a lyke case. S. Than must I borow 1 of the 3 comyng behynde next, and make that 0 to be 10, and then take I 2 oute of 10, and there resteth 8, and bycause I borowed 1 of the 3, I muste cancell the 3, and wrote 2 ouer it, than doothe the figure stande thus.

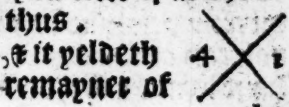
2 2
x x 6 3 8
x 3 6 x 8 0 (301
x 8 x 4 x
x

M. Nowe haue you doone, and yet remaineth 228, and youre quotient dooth shewe you, that yf you diuide 136280 by 452, you shall fynd your diuisor in your greater numbre 301, that is. CCC. tymes and ones, and 228 remaynynge. And in the other exaumple where I diuided 365 by 28, the quotient was 13, and 1 remained, wherby I know that in a yere (whiche conteyneth 365 daies) there are 13 monethes, reckenyng 28 daies (or 4 wekes) iuste to a moneth, and 1 daie more. S. Why than doo we call a yere but 12 monethes? M. Of that at a more conuenient tyme will I fully instruct you, but now it is not cõuenient to entangle your mynd with other thyngs thã do directly pertayne to your matter, therfore yf you can remembre what you haue herde, you haue lerned a short maner of diuision, which

I wolde haue you often to practise, so that you maie be perfect in it, and hereafter I will shew you certayn other prope poyntes touchynge it. S. Then I praie you yet telle me howe shall I examine and trie my woorke, whether I haue don well or no, that though no man be by me to tell me, yet I maie perceaue it my selfe. Mai. Some men (yea and cõmonly) woo trie that by the rule of 9, as in all the other kyndes, saue that their ordre is this. Fyrste they caste awaie 9 as often as thei can out of the diuisor, & that that remaineth, thei set at the one side of a crosse, as in our first example, the diuisor was 28, frome whiche you maie take 9 twyes, and 1 remaineth, which thei set by a crosse thus. Than doo they likewyse examyne the quotient (whiche in our example is 13) & frõ thens thei cast awai 9 as often as they can, & the remayner they set at the other side of the crosse, & than multiplie they togither those 2 remainers, & to it that amounteth, they adde the remainer of the diuisiõ, if there were any, frome that hole summe they withdrawe 9 as often as thei can, & the rest thei set at the head of the crosse, as in our example the quotient is 13, whiche maketh only 4, and therfore muste you sette 4 at the other syde of the crosse, thus.

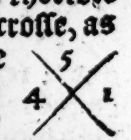
proufe.

Then multiply 4 by 1, & it yeldeth but 4, therto adde the remayner of

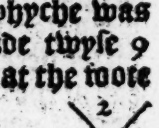


the

the diuision (which was 1) & it wil be 5, whiche sume wthe not amounte to 9, & therfore must be set holly at the head of the crosse, as you see here. And this nũbre on the head of the crosse is the first proue to which if you find an other lyke in ỹ numbre that was diuided, thã haue you done wel. Therfore now shal you likewise examin the hole summe that was diuided, & take awai 9 as often as you can, & that that remaineth, set at the foote of the crosse, & if it be equall to ỹ in the head of the crosse, than haue you well done, els not: as in our example the hole summe was 3, 6, 5, which maketh 14, frõ that take 9, & there resteth 5, which set at the foote of the crosse, thus. And you shal see ỹ they agree, therfore haue you done well. 3. Now wil I likewise examine our second example, where the diuisor was 4, 5, 2, which maketh 11, frõ thẽs I take 9, and the 2 that remaineth, I set at the right side of a crosse thus. Then examine I the quotiẽt, which was 3, 0, 1, where I find but only 4, that doo I set at the other side of the crosse, thus. Than doo I multiply 4 by 2, and it maketh 8, to that doo I adde the remainer of the diuision (which was 2, 2, 8, & maketh 12) and they two make 20, wherein I fynde twise 9 and 2 remaynyng, that 2 must I set at the hed of the crosse thus.



Than doo I examine the holle nũbre to bee dyuided, whyche was 136280, where I fynde twyse 9 and 2 remaynyng, whiche I set at the toote of the crosse, thus.



And bicause that it doothe agree with the figure at the head of the crosse, I knowe that the diuision was well wrought. M. This is the commen proue, how be it, the more certayne worckyng is by the contrary kynd, as to proue Diuision by Multiplication, thus. Multiplie the quotient by the Diuisour, and if the summe that amounteth bee equal to the sũme that shoulde bee diuided, then haue you well diuided, elles not. Howe be it, this must you marke, that if there remained any thyng after the diuision, that muste you adde to the sũme ỹ amounteth of the multiplication: as in our first example the quotiẽt was 13, and the diuisor was 28. Now multiply the one by the other, & the summe wil be 364, so that if you adde the 1 that remayned after the diuision, than will it be 365, which was the sum that shuld bee diuided, and therfore I knowe that I haue wel done. S. Now will I proue the same in the second example, whose diuisor was 452, and the quotient 301: these do I multiplie togyther, and there amounteth 136052, to which if I adde ỹ 228 that remained, shã wil it be 136280, which was ỹ hole

summe

summe to be diuided, & therfore I perceaue that I haue well done. M. This is the sureſt waie to examine Diuiſion by Multiplication, and contrary waies, the ſureſte proſe of Multiplication, is Diuiſion. And therefore now will I ſhewe how you maie proue Multiplication by Diuiſion When you haue ended Multiplication, and wolde know whether you haue well done or not, ſet the groſſe ſumme that amounteth of the multiplication ouermoſte, and diuide it by the multiplier, and if the quotient be the ſame numbre that ſhulde be multiplied, then haue you well wrought, els not: as in that example, where we multiplied 264, by 29, the groſſe ſumme was 7656. Now if you wyll know whether that multiplication be trew, you ſhall diuide that 7656, by the multiplier 29: And you ſhall perceaue that the quotient will be 264, and that is a token that you haue wel wrought. S, By your pacience I will proue that, and firſt I ſet downe the groſſe ſumme, and the multiplier not after the rule of Multiplication, but after the rule of Diuiſion, for now that numbre is become the diuiſor, that was before the multiplier, I ſhall ſet them therfore thus. 7656
And then ſhal I ſeke how many ti- 29
mes 2 in 7, that maie be 3 tymes and 1 remaineth, but then maie not 9 bee founde ſo often in 16, therefore muſte I take a leſſer

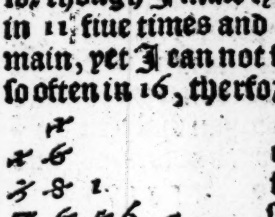
quotient, that is to saie 2, then saie I twise 2 maketh 4, whiche I take out of 7, & there remaineth 3, then doe I cancell 7 and 2, & ouer 7 I write 3, and in the quotient I set 2, so the figures stand thus.

3
Then saie I forthe, 2 times 7656 (2
9 make 18, whyche I bate 29
out of 36, and there resteth 18, then cancell I 3, and ouer hym set 1, and like waies I cancell 6 and 9, and ouer them I set 8, so ỳ thus stande the figures.

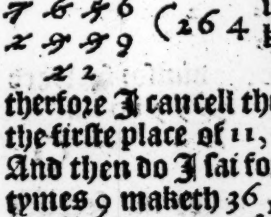
1
Then do I set forwarde the 38
diuisor by one place, and 7656 (2
seke a new quotient, that is 29
to saie, how many tymes 2 are in 18, which I fynde to be 9 tymes, but then can I not fynde 9 so many times in 5, therfore I take a lesser quotient, as to saie 8, but yet is that to greate, for if I take 8 tymes 2 out of 18, there remaineth but 2, & I can not fynde 8 tymes 9 in 25, therefore yet I take a lesser quotient, that is 7, which is also to greate, for if I take 7 tymes 2 out of 18, there resteth 4, but now I can not take 7 tymes 9 out of 45, therfore yet I seke a lesser quotient, as to saie, 6, the saie I 6 tymes 2 make 12, that I take out of 18, and there remaineth 6, so I cancel that 18 and the 2, and write 6 ouer 8, then saie I forthe: 6 tymes 9 maketh 54, that take I out of 65 and there remaineth 11, and the figures stande thus.

Then

Then must I set forth ye diui
for again, & then seke a new
quotient which will bee 4,
for though I make synde 2
in 11, fiue times and one re
main, yet I can not synd 9
so often in 16, therfore I set ye figures thus.



And the 4 in the quotient I multiply into the figures of the diuisor, saiynge, 4 tymes 2 maketh 8, which I take out of 11 & there resteth 3, therfore I cancell the 11 and ye 2, & set 3 ouer the firste place of 11, thus.



And then do I sai forth: 4 tymes 9 maketh 36, whiche I take from 36, and there remaineth nothyng, so that ye quotient of this diuision, where 7656 is diuided by 29, is 264, which doth declare ye if 264 be multiplied by 29, ye sume will be 7656. And thus I perceaue now how bothe Multplicatiō is proued by Diuision, & Diuision also by multiplication. M. Now haue I ended the .v. most commen kyndes of Arithmetike: for as touchyng Mediation, Duplation, Triplation & suche other, they are no seuerall kyndes of Arithmetike, but are contained vnder the other: for Mediatiō is conteined vn-

der

der Diuision, and is nothyng els but deuidyng by 2: and so are Duplation and Triplation contained vnder multiplication, for Duplatiō is nothing els but multiplyng by 2, & triplaciō is multiplyng by 3, of which I wil only propose examples, for the rules you haue herd alredy: if you wold mediat or diuide into 2, this sum 4531010, you shal set 2 for ye diuisor, & work as you lerned before, as thus

Then I fynde 2 in 4 two tymes, therfore my quotient must be 2, so I cancell 4 and 2, and remoue the diuisor forward thus.

4531010 (2
2

Then againe I fynde 2 in 5 twyse and 1 remaynynge, so I write 2 againe for my seconde numbre of the quotient, and cancel 5 and 2, and ouer 5 I set 1 thus.

4531010 (22
2 x

The remoue I the diuisor forward and seke a newe quotient, which is 6, then sale I 6 tymes 2 make 12, take that out of 13, and there resteth 1, so I cancell 2 and 13: & ouer 3 I set 1, thus.

4531010 (226
2 x x

Then remoue I the diuisor forward, and seke a newe quotient, which is 5, then take I twyse 5 out of 11, and there resteth 11, so I cancell the 2 and the laste figure of 2 and let the laste stande thus.

Then

Then remoue I the xx
diuisor forward and xxx 1010 (2265
seke a new quotient xxxx
whiche is 5, then take I 2 fyue tymes out of
10, and there resteth nothynge, then remoue
I againe the diuisor forward, thus.
But bicause I xx
can not fynde the xxx 1010 (22655
diuisor in ỹ num- xxxxx 2
bre ouer it, I must set a cipher in the quoti-
ent, and remoue the diuisor to the next place
thus. xx
Then seke I a xxx 1010 (226550
newe quotient xxxxxx 2
whiche I fynde to be 5, for so many tymes
maie I haue 2 in 10. Then haue I fully en-
ded this Mediation or Diuision by 2, and
the quotient is this, 2265505, whiche is the
halfe of 45310101, as you maie trye by Du-
plation: for double that quotient, or multi-
ply it by 2, and the same numbre wil amoũt.
But I wil no longer tary about these, seyng
they are but membres of the other kyndes.
But here now wil I teache you certaine easy Easy formes.
formes bothe of Multiplication and of Di-
uision, and fyrste of Multiplication. If you
wolde therefore multiply any summe by 10,
you shal nede to doe no more but adde a ci-
pher before his firste place: as for example:
36 multiplied by 10, make 360. Like waies
if you will multiply any summe by 100, put

two

two ciphers at his begynnynge. So if you wold multiply any sume by 1000, adde thre ciphers to the begynnyng of it. S. This doe I wel perceaue, and also the reason of it. M. I will omitte all reasons tyll our nexte metyng, when I shall tell you the reason of all other partes of Arithmetike also: and as to our matter nowe loke (as I haue tolde you) that you doe bothe remembre it, and also often practise it. But if you wold multiply any numbre by 5, marke firste whether the numbre be euen or odde, and if it be euen, take ỹ halfe of it, and write a cipher at the begynnyng of it, as for example: I wolde multiply 2564 by 5, I take the halfe of it, which is 1282 (as you maie know by Mediation) and before it I sette a cipher thus, 12820, & this is 5 tymes 2564. And thus maie you dooe with any other euen summe that you wolde multiply by 5. But if the summe bee odde, as for example: 2563, then muste you take the lesser halfe of it, or (if you will) take awaie 1 from the firste figure (as here from 3) and then take the halfe of the reste, and set before it 5: as of 2563, ỹ lesser halfe is 1281, for here I take but 1 for the halfe of 3, and if I putte 5 before that lesser halfe, then haue I multiplied it 5 tymes, as thus, 12815. S. What meane you by the lesser halfe? M. There is no iuste halfe of any odde numbre, therefore if we diuide an odde numbre into

two

two partes as nygh equall as can be, yet wil the one halfe exceade the other halfe by one, as for example: The two moste nereſt halfes of 9, are 5 and 4, and likewaies of 15, are 7 and 8, where you ſee that the one parte ſtyll is greater then the other by one. Now it is eaſy to know whych is the greater half, and which is the leſſer halfe. S. Then I perceaue you, and can doe lyke waies (I doubt not) with any ſumme: for if it be not beary eaſy to parte into halfe, then wil I doe it by Mediation eaſily enoughe. M, That is a ſure waie. And now haue you learned how to multiply eaſily by 5, 10, 100, 1000, and of lyke maner maie you doe with any other of that ſorte. But now if you will multiply by 20, 30, 40, and ſo forthe: or by 200, 300, and ſuche lyke, where there is one cypher in the firſt place, or made orderly in the fyrſte places, you ſhall take awaie thoſe cyphers. & multiply the ſumme oneli by the other figure or figures (if thei be many) and then at the begynnyng of the ſumme ſo amounteth ſhall you let ſo many ciphers as you toke awaye. Example of 2873, which I wolde multiply by 300. Firſt I caſt awaie ſ two ciphers frō the multiplier, and I multiply the ſumme by onely 3, that is lefte, and it amounteth to 8619, before whiche I putte the two ciphers, that I toke awaie before, and then is it 861900. And that is the ſumme that amounteth

mounteth, when 1873 is multiplied by 368. S. And if there were two or more figures beside the cyphers, I muste onely take awaie the cyphers, & multiply by the other figures, as I lerned before, as if I wolde multiplye 93648 by 25000, I shuld take awai the thre cyphers, & multiply the same by 25, and then at the begynnyng of that totall summe shuld I adde the thre ciphers againe. M. Euen so, but and if it chaunce the numbre that shulde be multiplied, or bothe the summes, as well the numbre that shulde be multiplied, as the multiplier, to haue ciphers in their first places, euermore caste awaye the cyphers and worke by the reste, but remembre to restore as many cyphers to the amountyng summe as you bated before, as in this exãple: 30200 shall be multiplied by 206, I shal onely take awaie the two ciphers frõ the greater numbre, and then multiply 302 by 206, and afterward adde the two ciphers againe. But if I wolde multiply the same 30200 by 2060, I shall not only take awaie the two ciphers frome the numbre that shulde be multiplied, but also I maie take awaie the one cypher from the multiplier, and then muste I adde 3 ciphers to the summe that amounteth: but take hede that you take awaie no cipher that commeth after any signifiyng figure, as in this laste example, you maie not take awaie that in the. iiii. place of the higher numbre,

nother

nother that in the thyrde place of the multiplier: howe be it, yet this you maye dooe, if one cyphre or more come in the myddes of your summes, you maie multiplie by the other figures, and ouerskyppe theim, but so, that you geue euery figure his dewe place, as thus: I wil multiplie 3026 by 2004, therfore I set theym thus.

3026
2004
24
1208

And thus I multiplie them: fyrst 4 tymes 6 make 24, I set the 4 vnder the fyrst place, and kepe the 2 in my mynde, or wryte it downe for easy remembrance: than saie I agayne: 4 tymes 2 maketh 8, and 4 tymes 0 maketh 0, then 4 tymes 3 maketh 12: but nowe wban I come to the nexte cyphre, bicause that it multiplieth nothyng, I let it go, and like waies the seconde cyphre: but then whan I doo come to the 2, and multiplie it into the 6 of the ouer numbre, you muste take heede (accordynge as I taughte you in Multiplication) that the fyrste numbre amountyng of the Multiplication, be set vnder the multiplier right, and the other ordrely toward the left hand, accordyng as you maie see in this example.

3026
2004
24

Where if you hadde expressed the cyphres after the common rate, than shoulde the fygures stande thus.

1208
12
604

But in effect all is one, saue that the fyrst waie by ouerskyppynge of the cyphres is the shorter and easier way: for that in effect they be bothe one, the addition of the parcels will declare, whiche in bothe wyll appere thus, 6064104, and nowe wyll I make an ende of this matter. Schol. Syr I thanke you: for I see greate ease in this waies of Multiplication, and if you can shew me such like in diuision, you shall greatly further me. M. Yes I wil teache you some easy waies in Diuisiõ also, & fyrst this: if you wold diuide any sume by 10, you shall onely with youre penne make a square line betweene the fyrst figure of your summe, & the second, & thã haue you done, for the holle numbre that foloweth ỹ lyne, stãdeth for ỹ quotient, & the figure ỹ is before ỹ lyne, is ỹ remainer, as for example: 3648 diuided by 10 wil stãde thus: 364|8 Where 364 is the quotient, and betokeneth that so many tymes ar 10 in 3648, and the 8 after the lyne is the remainer, whiche can not be diuided into 10, but by breakyng it into fractions, wherwith I wyl not meddle yet, and so lyke waies if you woulde diuide any summe by 100 with your penne, you shall cut awaie the 2 fyrst figures: and if you

3026
2004
24
1208
0000
0000
12
604

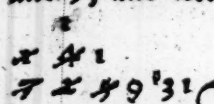
you woulde diuide by 1000, you muste cutte awaie the 3 first figures, and so of any other diuisor, whose last figure is 1, and the other be cyphres, looke howe many cyphres the diuisour hath, and so many figures at the begynnyng shall you cut awaie with the squier line, and they standeo alwaies for the remainer, bycause they are lesse then the diuisor, and can not bee diuided by it, and the other figures that bee behynde the lyne, stande for the quotient: But nowe if youre diuisour haue any other figure in his laste place than 1, and in all his other places haue cyphres, looke howe many cyphres there bee, cutte awaye so many of the fyrste figures of the numbre that shoulde bee diuided, and diuide the reste that followeth the lyne, by that figure that is in the laste place, as if it were the holle diuisour: exaumple of 6 4 2 8 4 whiche I wold diuide by 300, here must I cut away the two fyrst figures (for so many cyphres my diuisour hath) and must diuide the rest by 3, whyche is the figure in the last place of the diuisour. Fyrst therfore I part away the two fyrst figures, and the summe standeth thus, 642 |84. Than doo I diuide 642 by 3, and the quotient is 214, for in 6 I fynde twyse 3, and in 4 ones and 1 remaynynge, whyche 1 with the 2 nexte before dothe make 12, wherin I fynde 3 foure tymes, and this is a readye waye

to tourne shyllynges into poundes, for lyth 1 pounde dooeth conteyne 20 shyllynges, I muste diuide the holle numbre of shillynges by 20, therefore easyly to doo it, I see that my diuisour hath 1 cyphre, and therefore I cut awaie one figure from the begynnyng of the holle summe of shyllynges, and then I doo mediate or diuide by 2, the other figures or summe that foloweth. S. I will put an example: If I woulde diuide 64287 ℔, by 20, that is to saie: if I would tourne so many shillynges into poundes, I must cutte awaie the fyrst figure, that is 7, and diuide the reste, that is 6428 by 2, so shal the quotiēt be 3214, wherby I know, that 64287 ℔ make 3214 li. & 7 ℔ remainyng. M. Nowe proue by multiplication whether you haue well doon or no. Sco. The quotient is 3214. whiche I do multiplie by the diuisor 2, and it dooth amounte to 6428. Maist. Hereby maie you perceaue not onely that you haue well doon, but also how by diuision you may tourne shyllynges easyly into poundes. And contrary waies, by Multiplication you may tourne poundes into shillynges. But here shal you se amongest diuers men, diuers formes of such diuision, but if you marke what I haue tolde you, you shall perceaue easyly all their waies, for some men doo not cut awaie so many of the first figures of the sum that they wold diuide, as there are cyphres in the

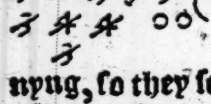
the

the firste places of their diuisor, but they set all their cyphres orderly vnder the fyrst places of the numbre that they woulde diuide, and then with the other figure (or figures yf they be many) they diuide the reste of theyr summe. Exaumple. If they woulde diuide 725931 by 3400, they sette theyr summes thus. 7 2 5 9 3 1 g000
And then do they diuide or- 3 4 0 0
derly tyll thei come to the cyphres: for there they staie and ende their woorke, as in this exaumple: They seeke howe often 3 maie be founde in 7, whiche is 2 tymes, and 1 remainyng therfore they set 2 in the quotient and cancell 3 and 7, and ouer 7 they set that 1 that remayned, thus. 1
Than doo they go foorthe, 7 2 5 9 3 1 (2
saiyng: 2 times 4 maketh 3 4 0 0
8, which they take out of 12, and there remaineth 4, thus. 4
Than renew they the di- 7 4 5 9 3 1 (2
uisor forwarde, and seke 3 4 0 0
howe often 3 maie be founde in 4, whiche is but ones, and 1 remayneth, than set they 1 in the quotient, & cancell 3 and 4, & ouer them thei set that 1, thus. 1
Than take they ones 4 4 4
oute of 15, and there re- 7 4 5 9 3 1 (21
steth 11: orelles more ea- 3 4 4 0 0
sily: take ones 4 out of 3
5, and there resteth 1, so they cancell the 4

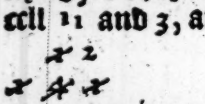
and 5, and sette 1 ouer theym thus.



Then set they foorthe the diuisor agayne, and seeke howe many tymes 3 are in 11, whiche they fynde 3 tymes and 2 remaynyng, so they set 3 in the quotient, and cancell 11 and 3, and ouer them setteth 2 thus.



Then doo they multiplie 4 by 3 which maketh 12, that withdraw they oute of 29, and there resteth 17, of whiche the 7 muste be sette ouer the 9, and the 1 ouer 2, thus.



And nowe are the 2 cyphers nexte ensuyng, so that the diuisor can no moze be sette forward, and therfoze is the diuision ended, and the remayner is 1731.

Nowe the quotient, which is 213, dooth declare, ỳ if you diuide 725931 by 3400, you shall fynde it therein 213 tymes, and there remaineth 1731, so shal you fynde it, if you worke as I taught you, by cuttyng away the 2 fyrst figures, bicause of the 2 cyphres, but this must you marke (as you may perceaue by this last exãple) that if there bee left any other remayner in the summe that was behynd the squire lyne, that remayner

must

muste be sette to the later ende of the fyrst remayner, whiche was cutte awaye with the squire line: as if you wold diuide 7 2 5 9 3 1 by 3 4 0 0, after the fourme that I taughte you, than would your summes appere thus.

```
    1
   1 1
 1 4 1 7
 7 2 5 9 | 31 (213
 2 4 4 4
   3 3
```

So that 17, which remaineth after the line must be sette to the 31 (that was cutte awaie with the line) in higher places, as you see here, where that 17 with the 31 do make 1 7 3 1. And here woulde I make an ende of Diuision, sauyng that there cometh to my mynde one late inuencion of easy Diuision, whiche I will brefely set forth to you, so that if you fynde ease in it, you maye vse it. Bycause that the hardest poynte in Diuisyon is the ready and easy findyng of the quotient numbre: and agayne, yf that be trewely knowen, all the reste is but lyght to be doone, therefore this waies shall you quickely and truely fynde the quotient. Fyrste write the. ix. figures of numbre, I meane 1 2 3 4 5 6 7 8 9, not alōg as I haue set them nowe, but vp & dowue, as in this fourme. And at the left syde of them drawe a longe lyne, as you see here; then cōsidre the diuisor by whiche you entende to woorke, and sette it on the lefte syde of the longe lyne

1
2
3
4
5
6
7
8
9

ryght agaynst 1, and for a distinction draw a line vnder it, then multiplie your diuisor orderly by eche of those figures, begynnyng with 2, and so go downewarde till you haue ended all, and looke what doothe amount of the multiplication of eche figure into the diuisour, that doo you write agaynst the figure, wherby you dyd multiplie. Scholar. By example I maie perceaue it better. Maister. Take this example, 263845 to bee diuided by 64, then muste I set the 9 figures, as I sayde before, and the diuisor muste I sette agaynst the 1, with a lyne drawen vnder, thus.

64	1
	2
	3
	4
	5
	6
	7
	8
	9

Than muste I multiply that diuisour by eche figure orderly: fyrste by 2, and it maketh 128, whiche I muste set agaynst 2 at the left hãde. Than multiplie I 64 by 3, and it maketh 192, whiche I set agaynste 3. Then 4 tymes 64 make 256, that sette I by 4. Than saie I: 5 tymes 64 make 320, that sette I agaynst 5. Than, 6 tymes 64 make 384, that sette I agaynst 6. Then 7 tymes 64 make 448, whyche I sette agaynste 7. Further I saye, 8 tymes 64, make 512, whyche I sette by 8. And laste of all I saye: 9 tymes 64, make 576, whyche I sette agaynste 9. And thanne they wyll stande thus,

292

And

And so is the table ended, by whiche you maie easily fynde the quotient, as you shall see by example now. Do you set downe the numbres (as you learned before) accordynge to the order of Diuision. S. That is thus. 263845

64

64	1
128	2
192	3
256	4
320	5
384	6
448	7
512	8
576	9

M. Nowe loke what numbre standeth ouer the diuisor, reckenyng therto all them that be behynde it towarde the left hande. S. Then are there ouer the diuisor 263. M. That is iuste. Now seke in the table on the left side, whether you cā fynd 263. S. It is not there. M. Then take that numbre that is next to it, benethe it: I meane a lesser numbre thē 263, but of all the lesser numbres that the table hath, take you that, that goeth nigheste to 263. S. That is 256. M. So is it: and marke this euer more, whē you can not finde iustely in the table that summe that is ouer your diuisor, then note that, that is next benethe it, of any summe that is in the table, & loke at the right hande of the lyne, what figure or diget that is against that summe, & take that diget for your quotient, and then worke on, as you lerned before: for now haue I tolde you the hole vse of this table. Howbe it, yet that you maie be sure to vnderstand it, I will se you ende this ersample of Diui-

sion by it. Now therfore begynne againe. 8. First I set downe the summes after the com men maner, thus.

2 6 3 8 4 5
6 4

Then doe I loke ouer the diuisor, and fynde there 263. Now to know how many times 64 maie be taken out of 263, I resorte to the table afore saied, and seke for the numbre 263, but it is not there, therfore (as you bad me) I take a lesser numbre, the nexte to it that I can fynde in the table, and that is 256, whiche numbre hath againste it on the right hande this digette 4, whiche I muste take for the firste figure of my quotient. Then doe I (as I learned before) multiply that quotient into euery figure of the diuisor orderly, withdrawyng the summe therof amountyng out of the ouer summe, as here I saie first, 4 tymes 6 make 24, so I take that out of 26 saiyng: 4 out of 6 remaineth 2, whiche I write ouer the 6, then 2 out of 2 remaineth nothynge, then cancell I 2 and 6, and also 6 in the diuisor, and the summe stande thus.

2
~~2~~ ~~6~~ 3 8 4 5 (4
~~6~~ 4

Then do I like waies saie forthe, 4 tymes 4 make 16, whiche I take out of 23, & there resteth 7 to be set ouer 3, and that 3 with the 2 behynde it, & the 4 vnder it must be cãcelled, as you se here. Thẽ haue I done with

~~2~~ 7
~~2~~ ~~6~~ ~~3~~ 8 4 5 (4
~~6~~ ~~4~~

the

the first figure of the quotient. M. Now set forward your diuisor, and seke a newe quotient, as you sought this.

S. Then thus standeth ỹ figures, so that ouer ỹ diuisor I se 78, which I seke in the table & can not fynde it, therfore I take the nexte lesser, & that is 64 the diuisor it selfe. M. So muste you doe when there is none other. S. Then against it I fynd this diget 1, which I must set in ỹ quotient before 4, thus.

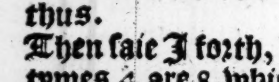
Then multiply I 6 by 1, and it is but 6 styll,

M. You nede not goe about to multiplie when the quotient is 1, for 1 doeth nother multiply nor diuide, but in such case onely subtracte the diuisor out of ỹ numbre that is ouer it. S. Then I take 4 out of 8, & there resteth 4, & 6 out of 7 there remaineth 1, so I cancelle those numbres & write ỹ remainers, ouer their places thus

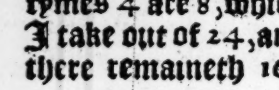
Thẽ set I forward ỹ diuisor againe, thus.

Where I se ouer ỹ diuisor 144, which I seke in the table & fynde it not, therfore I take ỹ numbre in ỹ table ỹ is next therto beneth it, whih I fid

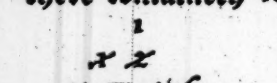
to be 128, againſte whyche in the right ſyde I fynde 2, whyche I take for my quotient, and that doe I multiply fyrſte into 6, and therof commeth 12, which I take out of 14, and then remaineth 2, that 2 I ſet ouer 4, and cancell the other figures, 1, 4, and 6, thus.



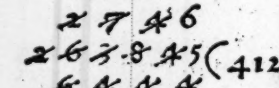
Then ſaie I forth, 2 tymes 4 are 8, which I take out of 24, and there remaineth 16,



of whyche I write the 6 ouer 4, & the 1 ouer 2, & cancel 2, 4, and 4, thus.



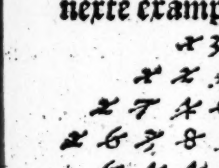
Now againe I ſet forwarde the diuiſor thus, And ſeyng ouer it 165, I ſeke that in the table, but fynde it not, therfore I take the nexte leſſer which is 128 againſt which I fynd 2, that do I ſet into the qnotient, & by it I multiply firſt 6, and therof cometh 12, which I take out of 16 and there reſteth 4, then cancell I 1, 6, and 6, & ouer 6 I ſet 4, thus.



Then doe I multiply 4 by 2, & it maketh 8 which I take

out

out of 45, and there remaineth 37, as in the nexte example folowyng.



And nowe haue I doone. M. Well, now I se ẏ you can worke by this kind of diuision, as farre forthe as I taught you. S. Yea syr, I thanke you, and I fynde in it much ease and certainnesse. M. Yet one thyng I doubt whether you perceaue: what if you did fynde in the table the numbre that standeth ouer the diuisor, what woulde you nexte doe? S. I thynke I shulde take the digette againste it on the lefte hande for the quotient. M. So is it: and as often as you seke in the table & fynde your numbre iuste, the digette againste it, is your true and iust quotient. I call that a true quotient, if it be the right quotient ẏ you shulde take, though your diuisor multiplied by the same, doe not clerely subtract the numbre ouer it, but there doeth somewhat remaine, as it chaunced in all the examples that you dyd worke by: but if it shulde chaunce (as it dooeth often) that your diuisour multiplied by your quotient, do subtract cleane the numbre ouer it, then call I that quotient not onely a trewe quotient, but also a iust quotient, bycause it doth iustly consume the nũbre ouer the diuisor: & that chaunceth euermore when the numbre ouer

ouer the diuisor is iustly found in the table. S. This I shall remembre. M. But yet one easy pointe more I will tell you in this sorte of Diuision, therfore marke it well: whē you haue found in the table, other ỹ same summe that is ouer the diuisor, other the nexte benethe (for lacke of the other) then loke what diget standeth against it, take that for your quotient, & bycause it is some paine to multiply the diuisor by the quotient, you shal not nede to doe it, but onely take the numbre that you founde in the table, & subtract that from the ouer numbre: for if you doe multiply the diuisor by the quotient, that will be the nūbre that shall amount, therfore is this waie more easier. S. So is it, and also more certainer, for suche as I am, ỹ might quickely erre in multiplyng, especially beyng smally practised therin. M. Thē proue in some breue example whether you can do it, and so will we make an ende. S. I wolde diuide 38468 by 24, therfore first I set the table thus. Then set I the two summes of diuision thus. And ouer the diuisor I fynd 38, whiche I seke in the table and fynde it not, therfore take I the next benethe it which the table hath, and that is 24 the diuisor it selfe, against which is set 1, which I take for ỹ quotient, which I set in his place.

38468	24	1
	48	2
24	72	3
	96	4
	120	5
	144	6
	168	7
	192	8
	216	9

and

And now I nede not to multiply the diuisor by it, but onely to withdraw the diuisor out of the 38 that is ouer it, and so remaineth 14, as thus.

Then set I forwarde the diuisor, and fynde ouer it 144, as appereth: then seke I that numbre in the table and fynde it, and againste it is 6, therfore I set 6 before 1, for my quotient, and I take that 144 for the iust multiplication of the diuisor by that quotient, and therfore without any newe multiplication, I do subtract that 144 from the other 144, and there resteth nothyng, as you maie se.

Therfore I set forwarde the diuisor, but seyng it will not be in the nexte place, (for then ouer 2 wold be nothyng) I set it forward twise, as you see here.

And for bicause that I coulde not set it in the nexte place folowyng, therfore I sette a cyphre in the quotient, as you se. Than loke I ouer the diuisour & finde 68, which I can not fynd in ♀ table, therfore take I ♀ next beneth it, which I fynd in the table, & that is 48, & agaynst it standeth 2, whiche I take for the quotient. And then without any multiplyng of the quotiente

uto

into the diuisor, I do subtract that 48 from 68, and there resteth 20, as here appereth.

```
1 4     2 0
3 8 4 6 8 (1602
2 4 4 2 4
  2
```

And so haue I ended the he holle diuision. Maist. Nowe can you sufficientely skyll in these kyndes of Arithmetike. And now for the vse of these two last, that is multiplication and Diuision, I wil brefely show you the feate of Reduction.

REDVCTION.

REductiō is, by which all summes of grosse denominatiō, may be tourned into summes of more subtil denomination: and contrary wayes: all summes of subtyle denomination maye bee broughte to summes of grosser denomination. S. What call you grosse denomination & subtile denomination? M. That I call a grosse denomination, which doeth contayne vnder it manye other subtiler or smaller: as a pound in respecte to shillynges is a grosse denomination, for it is greater then shyllynges, and conteyneth many of them. And shyllynges in comparyson to

Grosse denominatiō.

poundes

poundes, are a ſubtile denomination, for bi-
cauſe they are leſſer then poundes, and many
of them are cõteyned in one of the other: and
ſo lykewayes of other thynges, what ſo euer
thynge is compared to other, if it be greater,
and conteineth many of them, it is a groſſer
denomination: but if it be leſſer, ſo that ma
ny of theim are in the other, than are they
called ſubtile denominations: wherby you
may perceaue, that one denomination may
bee called a groſſe denomination, and alſo a
ſubtile (that is to ſaie, a great and ſmall) in
diuers comparisons. For ſhillynges compa-
red to poundes are a ſubtile or ſmalle deno-
mination, but compared to penies, they are
a groſſe or great denomination. Scho. Now
I vnderſtande the name, I praie you teache
me the vſe. M. The vſe is eaſyly learned, if
you remembre what you haue learned be-
fore. For if you will reduce any ſumme of a
groſſe denomination into a ſumme of a
ſmaller or ſubtiler denomination, you muſte
conſyder howe many of that ſubtiler deno-
mination do make one of the groſſer denomi
nation, & by that nũbre or numerator doo you
multipli the other ſumme: as if you wold reduce
20 poundes into ſhillynges, you muſt cõſider
that in a pound are included 20 ſhillings, ther
fore multiplie the one 20 by the other 20, &
there will amounte 400, whereby you maie
know, that in 20 pounde are conteined 400

Subtile & nominatiõ.

Groſſe and Subtile denominatiõ.

shillynges. Likewaies if you wold reduce 30 shillynges into penies, consydering that in 1 shillyng are 12 pennies, you must multiplie 30 by 12, and it will bee 360, whereby you fynd, that in 30 shillynges are conteined 360 pennies. And thus maie you reduce any gros denomination into a more subtiler, by multiplication, if you knowe howe many of the lesser doo make the greatter: of whiche thyng I will anone geue you a breue table for the moste accustomed kyndes of mony, weightes, measures and tyme, and suche lyke, wherby you may knowe how often eche subtile denomination is conteyned in the grosser, when you shal nede it, for the for saide kynde of reduction. And also the same shall serue you, if you wold reduce any sume of a subtiler denemination into a summe of a grosser denomination: For in suche reduction you must cōsider (as in the other form) howe many of the smaller do make the greater, and by that numbre must you diuide the other summe, and the quotient will declare how many of the greater denominuation are comprehended in that summe. as for example: If you wold know howe many shillynges are conteyned in 3240 penies, consider that 12 penies do make 1 shillyng, you must deuide that 3240 by 12, & your quotient will be 270, wherby you know that so many shillynges ar in 3240 pennies. But & you wolde

know

knowe farther howe many poundes are in thoſe 270 ſhillynges, ſeyng that euery pounde conteyneth 20 ſhillynges, diuide that 270 by 20, and it will be 13, and 10 remaynyng: wherby you maie know, that in 3240 pennies, or 270 ſhillynges, are 13 poundes & 10 ſhillynges: For euermore the remayner muſt be named, by ye name or denomination of the ſumme that was diuided, whyche in this place were ſhillynges. And thus maye you doo with any other kyndes of denominations. Wherefore to the intent that you may haue a light knowledge in the commen coynes, weightis, meaſures, and ſuch other, I haue prepared here a breefe table, whiche ſhall ſuffice to you at this tyme, tyl hereafter at more conuenient opportunitee I maye inſtructe you more exactely in the ſame.

A table for Englyſhe coynes.

Englyſhe coynes

A Souerayne.
Halfe a ſouerayne.
A rotall.
Halfe a rotall.
A quarter rotall.
An olde noble.
Halfe an olde noble.
An angell.
Halfe an angell.
A george noble.
Half george noble.
A quarter noble.
A crowne.
Halfe a crowne.
A crowne.
A grote.
A harpe grote.
A peny of 2 pences.
A dandy pratte.
A penny.
An halfe penny.
A farthyng.

 The penny

¶ The valewe of Englyshe coynes.

The valewe of englyshe coynes.

A Souerayn is the greatest englyshe coyne, and conteyneth 2 Rotalles or 3 Angelles, either 9 halfe crounes, or 4 crownes and an halfe, that is to saie 22 s 6 d.
Halfe a Souerayn is equall with a Rotall.
A Rotall conteyneth an Angell and halfe, that is to saie, 11 s, 3 d.
Halfe a Rotall conteineth 5 s 7 d ob.
A quarter of a Rotall conteineth 2 shillynges, 9 d, ob. q.
An olde Noble, called an Henry, is woorth 2 crounes, or a noble & halfe, that is 10 s.
Halfe an olde noble is worth 5 s.
An angell conteyneth a crowne and half, or 3 halfe crownes, that is 7 s 6 d.
Halfe an Angell is worth 3 s 9 d.
A Noble, called a George, is woorth 6 s 8 d
Halfe a Noble is woorth 3 s 4 d.
A quarter of a Noble (whiche in the old statutes is called a farthyng) conteineth 20 d.
A Crowne conteyneth 5 s, and the halfe crowne 2 s 6 d. Howbeit there is an other crowne of 4 s 6 d, whiche is knowen by the rose syde, for the rose hath no crowne ouer it, as in the other crowne, but it is enuironed on the 4 quarters, with 4 floure deluces, wherby you maie beste knowe it. But I will retourne to speake of the valewe of the coynes, for I entende not nowe to describe

the

the formes of them. Now of gold are there no more common coynes. In syluer, the greatest is a Grote, which conteineth 4 pe-nies. Thã is there an other Grote called an Harpe, whiche goeth for 3 d. Then next, is the penny of 2 d, and then a dandypratte worth thre halfe pennes. Nexte it a penny, then a halfe penny, and laste and least of all a farthyng, whose coyne is on the one syde a crosse, and on the other a purculles. This I tell you, bicause I see many y can not know a farthyng from a smalle halfepenny. Now haue I tolde you al the englishe coynes both of golde and syluer, but yet of the two most cõmen valewers of mony spake I nothing, that is to saie, of poundes and shyllynges, whiche though thei haue no coynes, yet is there no name more in vse than they, of whi-che the shillyng conteyneth 12 pennies or 3 grotes, y the poũd, 2 old Nobles, 3 George nobles, or 4 crounes, that is to saie 20 s. And this is now the rate of englishe monie. Here wolde I nowe expresse the valewes of sundrye other coynes of diuers countreies, but for three causes I nowe refrayne. The fyrst and chiefest is, bicause they ar not cur-rant by the statutes of this realme. An other cause is, by reason they are so vncertayne, that they be neuer longe at one rate. And a-gayne they are so different in so manny pla-ces, that it were matter inough for a greate

Syluer coi nes.

to speake sufficiently of theim all. How be it, yet because you shal not be all togither ignorant of them, I will shewe you the valewes of some that are moste in vse: and fyrste of Fraunce. The moste common mony are deniers, soulx, and frances, 12 deniers make 1 ß, 20 soulx make 1 franc, so that, as you see, these 3 kyndes are likein their rate to pennies, shillynges, and poundes with vs, but that this is the difference, that their denier is but the 9 part of our penny, and so their soulx (commenly called sowses) go 9 to our shillyng, and 9 of theyr frances to an englisch pounde of monye, so that 3 of their frances make a noble. And by those 3 may you practise howe to reduce Frenche mony into Englyshe mony. And as for the rest of their coines I will omitte tyl an other tyme, when I entende to shewe you the rate of sundry other kyndes of mony. But now as for the coynes of Flaunders bee so chaungeable, that you muste knowe them from tyme to tyme, or els you can not redute them into our mony certainly. But yet bicause that you shal haue an example of their mony to exercise you withall, you shal take those that be moste cõmon, as Stiuers, bothe single and double, grotes flämishe, carolus, and gyldens. A flemmishe grote is litteil aboue 3 farthynges englishe. A single stiuer is 1 d. ob. q. The double stiuer is 3 d ob. The syluer Carolus syngle 2 d ob. q. c.

Frenche coynes

Flaunders coynes

q̃.q.c. The double siluer Carolus, is 4 d̃, ob. q̃, q. Than is there also the Carolus gylden, whiche is worth 20 stiuers. And the flemmish noble is worth 3 Carolus gildens, and xii. stiuers. But I will let these passe now, exhorting you to practise to reduce those kindes into englishe money, accordynge as I haue set forth here folowyng: 2160 deniers make 240 d̃, or 20 s̃, 3240 deniers make 360 d̃, or 30 s̃: 8352 deniers make 928 d̃ or 3 li, 17 s̃ 4 d̃: 2160 soulx make 240 shillynges: and so of other the lyke rate. But if you wyl reduce flemmishe mony iustly, you must reduce it fyrst into the smallest parte of englishe money, that is in that coyne, as for exauuple. If you wold reduce 368 double stiuers into englishe mony, consideryng that a double stiuer conteyneth 3 d̃ q̃, you shall fyrst looke how many q̃ be in the double stiuer, and you shal fynd them 13, therfore multiplie the summe of the stiuers by 13, and thā haue you theie valewe in farthynges, which is 4784. Nowe if you diuide that by 4, then will there appere the numbre of pens, but better it were to diuide it by 48 (for so manye farthynges are in 1 shillynge) and then will the quotient declare the summe of shillynges. Lykewayes yf you woulde reduce any summe of syngle styuers into Englyshe money, you muste multiplie the sūme fyrst by 13, and than haue you the sum of q̃,

whiche summe if you diuide by 8, then will amount the sum of penies: or if you diuide it by 96, the summe of shyllynges will appere. But this marke in all diuision, whan ye do reduce to bring one denomination into an other, if there be any remayner after the diuision, that must be named by the denomination of the grosse sume that was diuided: as for example: I wolde bryng 254 q, into penys, therfore I do diuide that 254 by 4, for so many farthynges make 1 penny, and the quotient is 63, whiche is the summe of the pens, & then remaineth yet 2, whiche ar farthynges styll, as you may proue by diuidyng. And this must be marked in al diuisiō namely when it is done for reduction. And thus much haue I said of money, now wil I shew you in like sort the distinction of weighWeightes. tes, after the statutes of England, where the least portion of weight is cōmonly a grayn, A grayne: meanyng a grayne of corne or wheate, drie, and gathered out of the myddle of the eare. Of these graynes in tyme passed 32 waied iuste 1 penny of troye, and than was but 20 pennes in an ounce: But nowe ar there 46 pennes in an ounce, so that there are not fully 14 grayne in one penny. But an Vnce. nowe of ounces after troye rate whyche is a Penny of Troie. the standarde of Englande) 12 do make 1 pounde. But commonlye there is vsed Haberdepoise weyghtes. an other weyghte, called Haberdepoyse, in whiche

in which 16 vnces make a pound. Therfore when you woulde reduce vnces in to pounds, you muste consyder whether youre weightes be troy weightes or haberdiepoise, and if it be troie weight, you muste deuide your vnces by 12 to bryng them to poundes, but if it be haberdypoise, you muste diuide them by 16: now agayne there bee greater weightes, which are called a hundred, halfe hundred, and quarterne, and also halfe a quarterne. &c. S. Why? so there maie bee reckened 20 pounde, 40 pound, 200 pound, and suche vnnumerable. M. All these are numbers of weight, but thei haue not commen weightes made to their rate, as the other haue. And againe, these that I dydde name are not iust in numbre as thei seme by their name: for a hundred is not iuste 100, but is 112 pounde. And so the halfe hundred is 56, the quarter 28, and the halfe quarter 14: And this is the commen weightes vsed in moste thynges that are solde by weighte. How be it there are in some thynges other names: as in wolle, 28 pounde is not called a quarterne, but a todde: and the 14 pound is not named halfe quarterne, but a stone: and the 7 pounde, halfe a stone. Other names bycause thei differ in many places, and agree in fewe, I let them passe. But a sacke of wolle by the statutes is limitted to be 26 stone. Now in these though it be solde by the hun-

a Hundred weyght

Woulle weightes

a Todde

Stone

Sacke

These weightes

hundred, & by the stone in some places, yet the very weightes of it are cloues & weies, so that a cloue shoulde contayne 7 pounde: and awey 32 cloues, that is 224 pounde. Howe bee it some statute bokes saie that a cloue shulde be 6 pounde: and some saie also, that a weye dooeth conteine 36 cloues, and that is commenly vsed, for the common wey is of 256 pounde, that is 36 cloues, reckenynge 7 pounde to the cloue and there is 4 pounde ouer weight. And lette this suffice you at this tyme touchynge weightes. Now of weightes ar made other measures, bothe for grayne and lyquor. For a pounde in weighte, maketh a pinte in measure, so that 8 pounde (or 8 pyntes) doe make a galon: halfe a galon is named a pottell, and halfe a pottell is called a quarte, which containeth two pyntes. Now aboue a galon the nexte measure is a fyrken: then a tertian, a kylderkyn or halfe barrell, and a barrel. And by those measures are solde commenly Ale, bere, wyne and oyle, butter and lope, salmon herynges and eeles: but as these bee vnlyke thynges, so the measure of their vessels doe dyffer: for the measures of ale are as foloweth.

a Cloue Weye

Measures for lyquor

a Pynte Galon

Pottell Quarte Fyrkyn Tertian Kilderkyn Barrell

Ale measures

of ale	the fyrken ẏ kilderkē the barrel	containeth	8 16 32	galons.

of

of bere { the firken / ỹ kilderkē / the barrel } containeth { 9 / 18 / 36 } galons. Beere measures

Sope measures, bothe fyrken, kylderken, and barrell, shulde be all equall to Ale measures. Moreoner the statutes doth lympytte the weighte of euery of those thre vessels beyng empty. Sope measures

A barrell
Halfe barrel } to wey empty { 26 / 13 / 6ℓ. } pounde
A fyrken

Herynges also be sold by the same measures that ale and sope be solde by: but herynges also are solde by tale, 120 to the hundred, 10 M. to the last. Salmon and eeles, haue a greater measure. Herrynge. Salmō and Eeles.

Salmō & Eeles { the butte / ỹ barrell / halfe bar. / the fyrkē } holdeth { 84 / 42 / 21 / 10ℓ. } galons.

How be it, some statutes dyd limitte eele vessels equall with heryng vesselles. Now as for wyne vesselles seldome are smaller then hoggesheddes, whiche are of 63 galons: euery hogges hed is two barrelles: yet there are many other wyne vessels, but of them all se this table, and marke the measures one of an other. Wyne measures.

Of wine and oile.

the rondelet	holdeth	18 ℔.	Galons
the barrell		31 ℔	
ꝑ hogges hed		63	
the tertian		84	
the pype		126	
the tonne		252	

Tertians

But you shall marke that there be other kindes of tertians, for there bee tertians (that is to saie thirdles) of pypes, of hogges heddes, and of barels, as well of other thynges as of wyne. Also of malucseys and secke, &c. the halfe tonne is not called a pype, but

a Butte

rather a butte. And thus muche haue I thought mete to tel you at this time. S. And is this alwaies true? M. I haue tolde you how it shulde be, but how it is I maie not saie how thei doe differ daiely from their iust measure: the gagers can tel you better then I. But I will letre this passe now, & speake brefely of ꝑ other mesures. And as of weightes there dyd spryng the lyquyde measures wherof I spake laste: so of the same spryngeth dzie measures, as peckes, bushels, quarters, and such like, whereby are measured corne and lyke graynes: also salte, lyme, coles, and other lyke. And this is the ozder & quantitee of them.

Dry measures

A Pecke
A Bussheli
A Quarter
A Weye

A pecke is the measure of 2 gallons.
A bushell containeth 4 peckes.
A quarter holdeth 8 bushelles.
A weye containeth 6 quarters.

Theise

Theſe are the commen names and meaſures, but in diuers places there be diuers ſortes. And the buſhel in many places is 2 buſhels, but then is the buſhell there called a ſtryke. And in ſome places halfe a quarter is called a cornoke. But theſe diuerſitees are to many to tell you brefely them al. And agayn ſith thei ar againſt ỹ law & ſtatutes, I count them vnmete to be vſed. But now remaineth yet a nother kynde of meaſure, wherby men mete lengthe and bredth, and thoſe are an ynche, a foote, and ſuche other: whoſe names and lengthe, this table ſhoweth.

Strike

Cornoke

Meaſures to mete lengthe and breadthe

3 graines of barly make an ynche. An Inche
12 ynches make a foote. A Foote
3 foote make a yarde. Yarde
3 foote and 9 ynches make an elle. Elle.
5 yardes and halfe make a perche. Perche
1 perche in bredth, and 40 in lengthe doe make a rodde of londe, which ſome call a roode, ſome a yarde londe, & ſome a farthendele. Rodde Farthedele
2 farthendeles make halfe an acre of ground
4 farthendeles make an acre.

Here mought I tell you many thynges els touchyng meaſure: And alſo how to reduce ſtrauuge meaſures to our meaſures, but bycauſe it can not wel be done without the knowledge of fractions, whiche as yet you haue not learned, I will let them paſſe tyll an other tyme, when I ſhall enſtruct you in

Geo-

Geometry, wherin I shulde be enforsed els to repete the same often agayne. S. But yet for of the partes of tyme, I praie you tell me somewhat. M. You knowe that a naturall daie hath 24 houres, and euery houre hath 60 minutes, It nedeth not to tell you, that 7 daies make a weke, & 4 wekes, make a cõmẽ moneth, & 13 monethes make a yere, lacking 1 daie & certain howres and minutes: But of that I shall enstructe you hereafter. And here will I make an ende of Reduction for this tyme, whiche though it bee counted no kynd seueral of Arithmetike, yet you se it is no lesse nedeful to be knowen, nor easier to be done, then any of the other. S. Mary syr it semeth vnto me muche harder then any other sorte, for it requireth the knowledge of so many thynges. But now syr whẽ you see tyme I am redy to learne forth, for as much of Reductiõ as you haue taught me I remembre: but & if I doe at any time forget, I shal haue recourse to ỹ tables, whiche you haue set forth for me. M. So doe you, for it wyl not be remembred wout exercise. Now Progression I wyll begyn.

The partes of tyme a Day

a Houre Weeke Moneth, Yeere

PROGRESSION.

Progression Arithmeticall

PRogression is a breue and quicke addition of diuers summes procedyng by some certaine and reasonable order. I meane when the di-

staunce of euery 2 numbres is equall to the distaunce of the second numbre from the first S. I vnderstande you not well. M. By an example it will be plainer, as here: 1, 2, 3, 4, 5, 6, here you see the seconde to differ from the firste but by 1, and so doeth all the other, one excede an other by 1 styll to the ende. S. This I perceaue. M. And lyke waies here: 4, 7, 10, 13, 16, 19, 21, 24. S. Yea, thei procede by the difference of 3. M. And so I meane, that Progression is an art how to adde all such numbres and other like together, muche quicklyer then by commen Addition, and that by this meanes: Tell how many numbres there are, and if thei be odde, write theyr summe downe by it felse: as in this example, 2, 4, 6, 8, 10, 12, 14, where the numbres are 7, as you maie see, therfore set downe 7 in a place alone, as I haue done in the margent here: then adde together the firste numbre and the laste, as in this example: adde 2 to 14, and that maketh 16, take halfe of it and multiply by ẏ 7 whiche you noted for the numbre of the places, and the summe that amounteth is the summe of all those figures added together, as in this example: 8 multipled by 7, make 56, & ẏ is the sūme of all ẏ figures. S. That wil I proue by ā other exāple: I wold know how much this sūme is, 5, 8, 11, 14, 17 20, 23, 26, 29, I tel ẏ places, & thei ar 9, ẏ I note

Progression arithmetis coll

note: Then I put the first numbre 5, and the last 29 together, & they make 34, I take the halfe of it, that is 17, and multiply by 9, & it maketh 153. That you say is the summe of al the nũbres. M. So hal you findit, if you try it. S. How shall I trye it? M. By Additiõ: For if you adde al the parcels together you shall se the same summe amount, if you dyd worke well, and that Addition trieth all kyndes of progression. S. Then I can adde by Progression, if the nũbres of the parcels be odde. But what if thei be euen? as in this example, 1, 2, 3, 4, 5, 6, 7, 8, M. When the numbre of the parcels be euen, then note that also as you dyd before, and lykewaies adde the first summe to the laste, and by the halfe of the numbre of the places do you multiply it, as in your example, the parcels are 8, that note I, then addyng the first summe to the laste, there amounteth 9, that doe I multiplye by the halfe of parcelles, that is by 4, and it makethe 36, whyche is the summe of the 8 parcelles. But if you wyll take one rule for these bothe, do thus. Multiply the halfe of the one by the other hole, & the summe will amounte all one. For sometyme it chaunceth that the parcels be odde: so that their halfe can not be taken, and somtyme it chaunceth the addition of the fyrst numbre and the laste to be odde so that the halfe of it can not be taken: but they will neuer

will neuer be both odde. S. The I perceaue this, if there be no more longynge to it. M. This is inough for Progression Arithmeticall, howbeit, there is a nother maner of Progression called Geometricall, when the nũbres encrease by a lyke proportion, that is, if the seconde numbre containe the firste 2, 3, or 4 tymes and so forthe, then the thyrd containeth the seconde so many tymes also, and so the fourthe the thyrde, and the fifthe the fourth: wherfore I set those thre exãples. Here in the fyrst example 3, 6, 12, 24, 48, you se, that euery numbre 1, 3, 9, 27, 81, cõtaineth the other, (that 2, 10, 50, 250, goeth nerte before hym) 2 tymes: and in the seconde example 3 tymes: in the thyrde example 5 tymes. Now if you will know how to fynd easily the summe of any suche numbres, doe thus, consydre by what numbre thei bee multiplied, whether by 2, 3, 4, 5, or any other, and by the same numbre doe you multiply the last summe in the Progression. S. I praie you worke it by this example, 2, 8, 32, 128, 512, 2048, which summe I haue encreased by the multiplication into 4. M. Then muste I multiply the last sume (whiche is 2048) by 4 also, and it will bee 8192. Now muste I bate from this summe the fyrste numbre of the Progression, which here is 2, then resteth 8190, which summe I muste diuide by 1 lesse then was the num-

Geometricall

ber that I multiplied by. Seyng thā I multiplied by 4, I must diuide by 3, so diuidyng 8190 by 3, the quotient will be 2730, whiche is the summe of all the Progression. And now to proue whether you can doo the same I geue you these numbres to adde, 3, 15, 75, 375, 1875, 9375, 46875. &c. I can not well telle by what numbre this progression dooeth encreace. Mai. In any suche doubte doo thus: Diuide the second numbre by the fyrst, and the quotient wil shew you the numbre that engendreth the progression. Schol. Then is that numbre in this example 5, for so many tymes is three in 15. M. So is it. Nowe worke as I taught you. S. The laste numbre is 46875, which I multiplie by 5, and it yeldeth 234375, from whiche I bate the first numbre of the progression, that is 3, and there resteth 234372, which I diuide by 4, for that is 1 lesse thā 5, & the quotient is 58593, which is the holle sum of the progression. M. Now if you remēbre wel this, you haue learned the art of progressiō both Arithmetically, & also Geometrically: which you maie proue other by subtraction of eche numbre alone frō the sūme, & so wil there nothyng remain, other by addyng togither of all ỹ parcels, for so wil the same summe amount. And now for the vse of this rule, I wil put forthe to you certayne questions, whiche some doo referre theim to Addition, but not so iustely

as

as I doo vnto this rule of progression: and some as vncircumspectely referre the same to Duplation.

The fyrst question is this.

If I solde vnto you a horse, haunge 4 shoes, and in euery shoe 6 nayles, with this condition, that you shall paie for the fyrste nayle 1 ob. for the second 2 ob. for the third 4, and so foorth, doublyng vnto the ende of all the nayles. Nowe I aske you howe muche woulde the holle price of the horse come vnto. Sco. Fyrst to know the numbre of the nayles, I must multiplie 6 by 4, & that maketh 24, then I will doo thus. I will write the numbre of the nayles euery one in order from 1 to 24, and agaynste eche numbre of nayles the summe of halfe penies dewly, as the order of Duplation teacheth, and in this nexte figure folowynge, appereth. Questis of sellyng a horse.

Than do I resort to the rule of Progression, where I cõsider, that the encreace of this summe procedeth by multiplication of 2, and therfore I do multiply the last sum by 2 also, and it yeldeth 16777216, from which I abate the first numbre, which is 1, & then resteth 16777215, whiche I shulde diuide by 1 lesse, than I dyd multiply. But seyng that it is 1, I nede not to diuide it, for 1 (as I haue before sayd) dothe nother multiply nor diuide: therfore I take that sum 16777215 for the wholle summe of the halfe pennyes,

1	1
2	2
4	3
8	4
16	5
32	6
64	7
128	8
256	9
512	10
1024	11
2048	12
4096	13
8192	14
16384	15
32768	16
65536	17
131072	18
262144	19
524288	20
1048576	21
2097152	22
4194304	23
8388608	24

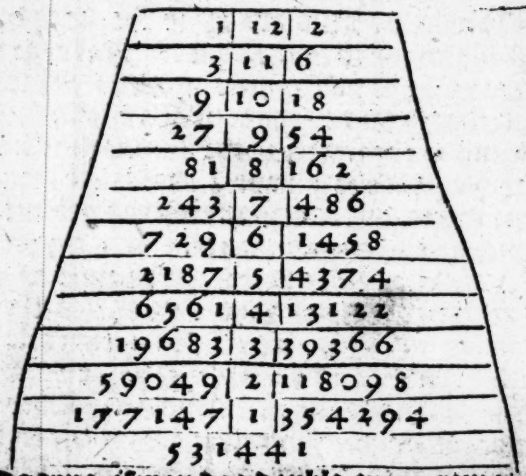
whiche by reduction I fynde to be 6 99050 li, and 7 d, ob. that is, 3495 li, 5s li, 7 d, ob. Maister. That is well doon: but I think you wyll bye no hoyse of the price. Scholar. Mo syr, yf I be wyse. Maister. Welle, then aunswere me to this questyon.

A lorde delyuered to a brycklayer, a certayne number of lodes of bryche, whereof he willed hym to make 12 walles, of suche sorte that the fyrste walle shoulde receaue 2 thyrdeles of the holle numbre: and the seconde 2 thyrdeles of that, that was lefte: and so euery other 2 thyrdeles of that, that remayned: and so dydde the brycklayer. And lohanne the 12 walles were made, there remayneth one lode of bryche.

Question of bricke laying

Nowe I aske you howe many lode went to euery walle, and howe many lode was in

in the wholle? Scholar. Why syr, it is vnpossible for me to tell. Maister. Naye, it is verye easye yf you marke it well: Marke well, that I sayde, that euery walle shoulde receaue 2 thyrdeles of the summe that was lefte. Nowe take awaye 2 thyrdeles frome any summe, and you muste needes graunte, that that whyche remayneth, is 1 thyrdele of the summe laste before: exaumple of 9, frome whyche if you take 2 thyrdeles, there wylle remayne 3, whyche is 1 thyrdele of 9: lykewaies from 3 bate 2 thyrdeles, and there wyll remayne 1. Scholar. This is trewe, and nowe I perceaue that the leaste walle hadde but two lode of brycke. Maister. And by the same reason may you knowe howe many lode euery walle hadde, accordyng as this fygure folowyng dooth shew, and lykewayes what the whole summe of bryckes was: for yf you make 12 summes multiplyinge by 3 styll, frome the laste remayner, as you see here on the lefte syde of the table, there wyll appere all the remayners after euery walle: and yf you multiply the laste of those 12 summes by 3 also, then wyll that be the summe of the lodes, whyche were delyuered to the bryckelayer.



1	12	2
3	11	6
9	10	18
27	9	54
81	8	162
243	7	486
729	6	1458
2187	5	4374
6561	4	13122
19683	3	39366
59049	2	118098
177147	1	354294
531441		

Agayne, if you doo double euery remainer, as you se at the right side of this table, those numbres will shewe the summe of lodes that went to eche wall: wherby also you may perceaue, ꝑ eche wal was 3 tymes so great as ꝑ nexte lesser. S. Lo, nowe it appereth easy ynough. Now surely I see that Arithmetike is a right excellent arte. Mai. You wil saie so, when you knowe more of the vse of it: For this is nothyng in comparison to other poyntes that maie be wrought by it. Scolar. Then I beseeche you syr, ceasse not to instruct me farther in this wonderful cunnyng Maister. By the order of the science (as men haue taught it) there shuld folowe next, the Extraction of rootes of numbre, whiche bycause it is somewhat harde for you, yet

The rule of proportiõs

yet I will lette it passe for a whyle, and will teache you the feate of the rule of proportions, whiche for his excellencie is called the Golden rule. Whose vse is by 3 numbres knowen, to fynde out an other vnknowen, whiche you desyre to know, as thus: If you paie for your boord for thre moneths 16 ℔, how muche shal you paie for 8 monthes? To know this & all such like questions, you shall consider which two of your 3 numbres be of one denomination, and set those 2, the one ouer the other, so that the vndermoste be it, that the question is asked of: As in my question 3 and 8 be both of one denomination, for thei bothe be monthes: & bicause 8 is the numbre that the quotient is asked of, I sette then one ouer thother, & 8 vndermost, thus, 3 with suche a crooked draught of lines. 8 Then doo I set the other numbre, whi che is 16, agaynst 3, at the right syde of the lyne thus. 3 16 8 And now to know my question, this must I doo: I must multiplie the lowermost on the left side by that on the right side, & the sume that amounteth, I must diuide by the hyest on the left syde: Or in plainer wordes thus: I shall multiplye the noumbre, of whyche the question is asked (whyche is called the thyrde number) by the numbre of an other denomination (whiche is called the second) and that summe that amounteth, muste I

The golden rule

Questis bordyng

The thirde numbre

The seconde numbre

The fyrste numbre.

diuide by the summe of like denomination, which is called the fyrst. Than for the knowledge of this question, I multiplye 8 into 16, and there amounteth 128, whiche I diuide by 3, and it yeldeth 42 ℔, and 2 ℔ remayneth, whiche I tourne into penies, and they be 24 ð, of whiche the thyrde parte is 8 ð: so the thyrd part of 128 ℔, is 42 ℔ 8 ð, whiche summe I write at the right hand of the figure against 8, thus. 3 — 16 ℔.

And hereby I know, ℈ if 8 — 42 ℔, 8 ð, thre monethes boordyng do come to 16 ℔, that 8 monethes boordynge wille come to 42 ℔ 8 ð, and like waies of any other lyke question. But here must you marke, that the fyrst numbre and the thyrde be of one denominatiõ, and also the seconde & the fourth, for whiche you seeke: or els be of suche denominations, that you in woorkynge may bryng them into one. As if a man shuld aske me this question: 12 wekes tourneiyng cost me 14 nobles, howe many poundes is that in one yere? Here you see no 2 numbres of one denominatiõ, but yet in woorkynge you maie tourne them into lyke denominations, as thus: Tourne the one yere into 52 weekes, and the fourthe summe wyll bee nobles, by the order of the woorkyng. Then to knowe this question, multiply the third sume 52 by the second 14, and the summe will be 728, that diuide by 12, and it will be 60,

Questiõs of expences

and

and 8 remainyng, whiche if you tourne into ſhillynges, they wil be 53 ß 4 d, which if it be diuided by 12, wil yeld 4 ß 5 d, and the thirde parte of a penny: put this 60 nobles (which maketh 20 li) with the 4 ß 5 d, and q, and littell more: for the ſumme that anſwereth to the queſtion, and it is the expence of a yere, and the ſummes will be thus.

12 — 14 nobles
52 — 60, 4 ß 5 d q.

And take this for a generall rule, that euermore the thirde numbre be it, that the queſtion is ioyned with: and the firſt, the numbre that is of the ſame denomination, then muſt the ſecõd nedes be that other. And remembre alſo, that the place of the firſte numbre is the higheſt on the lefte ſyde: and the place of the ſeconde ryghte agaynſt it on the ryght ſyde: the place of the thyrd numbre is vnder the firſt, as by thoſe examples you haue ſene. S. This I truſt I can doo. Maiſt. But and the queſtion be aſked thus: In 8 weekes I ſpend 40 ß, how longe will 105 ß ſerue me? Though the order ſeeme vnlyke, yet take you 105 for the thyrde numbre, and 40 beyng of the ſame denominatiõ for the firſt, & then 8 for the ſecond. Then multiply 105 by 8, and it will be 840, whiche if you diuide by 40, it wil yeld 21, which is the fourth numbre, and ſheweth howe many weekes 105 ß will ſerue, if you ſpend 40 ß in 8 wekes. The figure of this

A generall rule.

Queſtiõ of expences.

queſtion is this, as if you ſhuld ſaie: if 40 s. ſerue for 8 wekes, 105 ſerue for 21 wekes. Other diuerſitees there be of worckyng by this rule, but I had leauer that you wolde learne this one well, then at the begynnyng to trouble your mind with many formes of worckynge, ſith this waie can doe as muche as all the other, and hereafter you ſhal learne the other more conueniently. But yet before we make an ende of this rule, this ſhall you note, that there is an other order quite contrary to this that you haue learned. For in this rule hitherto, euermore loke how much the thyrde numbre is greater then the firſte, ſo much the fourth numbre is greater then the ſecond: and contrary waies, loke how much the firſt ſumme is greater then the thyrde (if it doe chaunce ſo) ſo muche is the ſeconde ſumme greater then the fourth. But there is a contrary order, as this: That the greater the thirde ſumme is aboue the firſte, the leſſer the fourth ſumme is benethe the ſecond: and this rule you maie call the backer rule: as in example. If I haue bought 30 yardes of clothe, of 2 yardes bredth, and wold dye canwas of 3 yardes brode to lyne it withall, how many yardes ſhuld I nede? S. Why? there is none ſo brode. M. I doe not care for that, I doe put this example onely for your eaſy vnderſtandyng: For if I ſhulde put the example in other

40 — 8
105 — 21

The Backer rule

Queſtiõ of dyeng of clothe

other measures, it wold be harder to vnderstande: but now to the matter. If you wold knowe this question, sette your numbres as you dyd before: but you shall multiply now the firste numbre by the seconde, and that ariseth therof, you shall diuide by the thyrde, whiche thyng if you doe here, I meane, if you multiply 30 by 2, it will be 60: whiche summe if you diuide by 3, there will appere 20, wherby I know, ꝑ if 30 yardes of clothe of 2 yardes brode shulde be lyned with canwas of 3 yardes brode, 20 yardes of canwas wold suffice: as this figure sheweth.

And now bycause you founde fault at my example, how saie you, perceaue you this? S. Yea syr, I suppose

2 — 30
3 — 20

M. Then answere me to this question: How many elles of canwas of elle bredthe, wyll serue to lyne 20 yardes of sey, of 3 quarters of a yarde brode? S. In good faith syr I can not tell, for I knowe not how to bryng the summes to lyke denominatiõs. M. Then I will tell you, sith there is mention here of quarters, & again euery one of ꝑ mesurs both elles & yardes maie be parted into quarters, do you parte them so, bothe in the bredth & length, & then put forth the question by quarters. S. Then I shall saie thus: how many quarters of canwas of 5 quarters brode will line 80 quarters of 3 quarters brode. M. now ãswere to ꝑ question. S. First I wil set them

Another like questiõ

downe in their forme thus: for 5 is ioined with the question, and is therfore the thyrde numbre, then is 3 the numbre of the same denomination, I mean because thei be both referred to bredth Now I multiply 80 by 3, and it is 240, whiche I diuide by 5, and it yeldeth 48. Then saie I, that 48 quarters of 5 quarters brode will suffise to lyne 80 quarters of 3 quarters brode. M. Turne the quarters againe into elles & yardes. S. Then I saie, that 9 elles and thre quarters of a yarde, of elle brode, will serue to lyne 20 yardes of 3 quarters brode, as this figure sheweth.

3 — 80
5 — 48

M. This rule is so profitable for all estates of men, that for this rule onely (if there were no more but it) all men were bounde hyghly to esteme Arithmetike. By this rule maie a capitaine in warre worke many thynges, as I will hereafter enstructe you abundantly: onely now I will shew you this one example. If it shulde chaunce a capitaine, whiche hath 40000 sowldiours to be so enclosed with his ennemie, ẏ he coulde haue no fresshe purueiance of vitailes, & that ẏ vitailes which he hath, wold serue ẏ army but onely 3 monethes, how many mē shuld he dymisse to make ẏ vitaile to suffice the residu 8 monethes? S. As you taught me, I set ẏ numbres thus: saiyng: If 3 monethes suffice 40000, to

Questiō of prouision touchynge au armye

3 — 40000
8 —

how

how many will 8 monethes suffice? to know this I multiply the first nũbre 3 into ỹ secõd 40000, and it yeldeth 120000, which same, I diuide by 8, and there will be in the quotient 15000, whiche if I doo subtracte from 40000, the remainer will declare that he muste dimisse 25000, as this figure sheweth.

3 — 40000
8 — 15000

M. Well, sithe you perceaue now the vse of this rule, I wil shewe other, which ensewe of the same. And firste the double rule, whiche is so called bycause there is in it double worckyng, by whiche thynge onely it differeth from this. S. Then by an example I shall vnderstande it well enough. M. So shall you, and let this be the example. If the cariage of 100 pound weighte 30 myles, doe coste 12 ō, how much will the cariage of 500 weighte coste, beyng caried 100 myles? S. I praie you showe me the worckyng of it. M. You must make two worckynges of it: the firste thus. If 100 pound weight coste 12 ō. how much will 500 pound coste? Set your figure thus, and multiply 500 by 12, and therof amounteth 6000, which if you diuide by 100, the quotient will be 60. that is the pryce of 500 for 30 myles. Then begyn the seconde worck, saiyng: if 30 myles cost 60 ō. how muche will 100 myles coste? Set your figure thus. And then multiply 100 by 60,

The double rule.

100 — 12
500 —

30 — 60
100 —

where

wherof amounteth 6000, whiche beyng diuided by 30 will yelde 200. Then you maie saie that so many pennies shall coste the carriage of 500 pound weight, 100 myles after ỹ rate of 12 d, for the 100 caried 30 myles. S. Now I perceaue it also. M. Then answere me to this question, 30 bushels of wheate sowed, yeldeth in one yere 360, how many wil 80 bushels yeld in 7 yere? I meane sowing euery yere of those 7 syth 80 bushelles. S. Fyrste I saie, that if 30 bushels yelde 360 in one yeere, then 80 bushelles wyll yelde 960, in one yeare. Then for the seconde worke, I saie: if one yere yelde 960, then 7 yeare will yelde 6720, as these two figures doe shewe.

Questis of sowyng

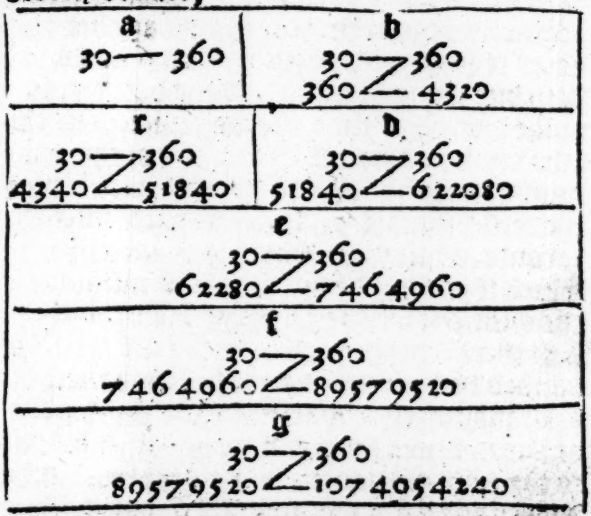
30 — 360 1 — 960
80 — 960 7 — 6720

Question of corne

But now syr, if I sette forthe 30 bushels of corne to an other man for 7 yeare, agreyng so, that shal he sow euery yere ỹ hole encrease of the corne, and I at the ende of those 7 yeares to haue the halfe of the hole encrease, I wold know how many bushels wyll there amount to my parte, supposyng the encrease to be after the rate of the laste question, for 30 bushelles in 1 yere 360? M. In suche a question you must haue so many seueral workynges, as there be yeres: as for example. In the fyrste yeare 30 bushelles yelde 360, then to know the yeldyng of the second yere, I muste saie if 30 yelde 360, how many yeldeth

deth 360? Worke by your rule, and you shal fynde 4320. Then saie for the thyrde yere: if 30 yelde 360, how many will 4320 yelde? you shall haue 51840, and so euery yeare, multipliyng the hole encrease by 360, and deuidyng it by 30, the encreace of the nexte yeare will amount, as these 7 figures dooe orderly declare,

a 30 — 360	b 30 — 360 360 — 4320
c 30 — 360 4340 — 51840	d 30 — 360 51840 — 622080
e 30 — 360 62280 — 7464960	
f 30 — 360 7464960 — 89579520	
g 30 — 360 89579520 — 1074054240	



where I haue set 7 letters for the 7 yeares, of whiche the fyrste is set without arte, bycause that is the encrease whiche you dooe presuppose, and the laste numbre of eche other doeth show the encrease of the yeare ÿ it standeth for, which ÿ letters doeth declare, so that

that the encreaſe of the 7 yeare is 107495 4240 buſſhelles: howe many quarters that is, and alſo howe many weis, you maie by Reduction ſone fynde. Now with one queſtion more I will proue you. If 6 mowiers doe mowe 45 acres in 5 daies, howe many mowiers will mowe 300 acres in 6 daies?

Queſtiō of mowyng

S. If 45 acres do requyre 6 mowiers, then 300 acres requireth 40. Nowe againe if 5 daies require 40 mowyers, then 6 daies nedeth but 33 mowiers. M. Why doe you not make mentiō of the 2 that remaineth in the laſte diuiſion? for the laſte parte of the queſtion is wrought by the backer rule, where the fyrſte numbre 5, is multiplied into the ſeconde, that is 40, wherof amounteth 200, which if you diuide by the thirde numbre 6, the quotient will be 33, as you ſaied, but the will there remaine 2, which can not well bee diuided into 6 partes: howbeit you maie vnderſtande by the 6 parte of 2, the thyrd parte of one mannes worke, whiche you muſt put to the 33: or els you maie ſaie that 33 workmen wyll ende all the 300 acres in 6 daies, ſaue two mens worke for one daie, or two daies worke for one man. But ſuche broken numbre called fractions, you ſhal hereafter more better perceaue when I ſhall holly enſtructe you of them. But now will I ſhowe you of the rule of felowſhip or company, which hath ſundry operations accordyng to the diuers

Rule of felowſhyppe

numbre

numbre of the company. This rule is some tyme without difference of tyme, and sometymes there is in it difference of tyme. First I will speake of that without difference of tyme, of whiche lette this be an exaumple. Foure marchantes of one company made a banke of mony diuersly, for the fyrst layd in 30 li, the seconde 50 li, the third 60 li, and the fourth 100 li, whiche stocke they occupied so longe, tyll it was encreased to 3000 li. Nowe I demaund of you, what shulde eche man receaue at the partyng of this monye? S. I perceue that this rule is like the other, but yet there is a difference, which I perceue not. M. Then wil I shew it to you. Fyrst by addition you shall bryng al the particular summes of the marchauntes into one summe, whiche shalbe the fyrst summe in your woorkyng by the golden rule, and the wholle sum of the gaynes with the stocke shall be the second summe. Now for the thyrd summe, you shall set the portion of eche man one after an other, and then worke by the golden rule, & the fourth summe will shewe you eche mans gains: as in example. The parcels of those foure marchantes make in one summe 240 li, set that in the fyrste place, the gaynes in the second, and the fyrste mans portion of stocke in the thyrde place, thus.

Without tyme.

Questiō of a bancke

240 — 3000
30 —

Now multiplie the second by the thyrde, and it wyll bee

90000, whiche you shall diuide by 240, and there will appere 375 li, thus,

240 — 3000
30 — 375

and that is the gains for the fyrst man. Now for the secõd man set the 50 li. that he brought in the third place, and worke as before, and his parte will be 625 li, as this figure showeth.

240 — 3000
50 — 625

Likewaies for the thyrd man set his mony, which was 60 li, and his part of gaines wil be 750 li, as here appereth.

240 — 3000
60 — 750

And so forth fourth mã, if you set his summe whiche is 100 li, his gains wil be 1250 li. as the proue wil declare:

240 — 3000
100 — 1250

S. This I perceaue, but is there any wai to examin whether I haue well done or no? M. That muste you doo by one cõmon proue, whiche serueth to the golden rule, and all other ensuyng of the same: And that is this. Change the standynges of the numbres, and set the thyrd in the first place, the fourth in the second place, and the first in the third place, and thã worke by the golden rule, and if you haue don well, the fourth numbre now wil be the same, that was the second before. As for exãple, I will take the laste worke whiche was this:

Proue.

240 — 3000
100 — 1250

which to examine, I alter as I saide thus.

100 — 1250
240 —

Nowe if I multiplie the second numbre by the thirde, and diuide that, that

40 40 40
28 28 28
12 12 12

that amounteth by the firſt, the will the fourth numbre be 3000, which was the ſecond before, as you ſee here. 100 — 1250 / 240 — 3000

whiche is a token, that I haue wel doone. But as in a ſyngle rule, one proſe thus is ſufficient: ſo in a rule where many operations be, you muſte tourne euery of them as I haue doon with this one. Scolar. Then for the proſe of the firſt worke of this rule I ſhoulde tourne the numbres thus.

30 — 375 / 240 —

And for the ſeconde thus.

50 — 625 / 240 —

And for the thirde thus.

60 — 750 / 240 —

And in eche of theim, if the worckyng wer true, the fourth numbre will be ſtyll 3000. Mai. Well, nowe an other exaumple will I put to you, not of gaines, but of loſſe: for one reaſon ſerueth for both. If thre marchauntes in one chippe, and of one felowſhip, had bought marchandiſe, ſo that the firſt had laide out 200 li. the ſecond 300 li, and the thirde 500 li, and it chaunced by tempeſt that they dyd caſt ouer bord into the ſea marchandiſe of the valew of 100 poũd, how much ſhuld eche mã beare in this loſſe? S. If I ſhall dooe in this as you dyd in the other queſtion, then muſt I ioyne their thre portions togither, 200, 300, & 500, whiche maketh 1000: than ſaie I, if 1000 leſe 100, then ſhall 200 leſe 20, and 300 ſhall leſe 30, and 500 ſhal leſe 50, as by theſe. iij. figures it doth appere playne.

Question of loſſe.

1000 — 100 1000 — 100
200 — 20 300 — 30
1000 — 100
500 — 50.

Mai. Thus you perceaue the vse of the rule without tyme. And that you maie as welle perceaue the same with diuersitee of tyme, I propose this example. iiij marchantes made a common stocke, whiche at the yeres ende was encreased to 35145 li. Now to knowe what shall be eche mans portion of gaynes, you must knowe eche mans stocke and tyme of continuance. The fyrst man of these. iiij layde in 669 li, which he dyd take from the stocke agayne at the ende of 10 monethes. The seconde man layd in 810 li for 8 monethes. The thirde layde in 900 li, for 7 monethes. And the fourth layde in 1040 li, for 12 monethes. This question shall you examine as you did the other before, sauyng that where as in the thyrde place of the figure you set eche mans summe alone, here you shall set the same beynge multiplied by the number of their tyme: and lyke waies in the fyrst place of the figure, you shall set their holle summes so multiplied by theyr tyme, and added into one summe, as thus: The fyrst mans summe is 669 li, which I multiplie by 10 (that was the number of his tyme) and it maketh 6690. The seconde mannes summe 810 li, multiplied by 8 (which was his

The rule of felowshyp With tyme

Question of chaunge

his tyme) maketh 6480. The thirde mans summe 900 li, multiplied by 7 (for that was his tyme) yeldeth 6300. The fourth mans summe was 1040 li, and his tyme 12, multiply the one by the other, & it will be 12480. These .iiii. summes thus multiplied by their tyme, must be set orderly in the thyrde place of the figure, & in the first place must be sette the holle summe of all .iiii. whiche is 31950. Now to end the questiō, I sai fyrst: if 31950 did get 35145, what dyd 6690 get? Answer 7359 li, as by this figure appeereth.

a

31950 — 35145
6690 — 7359

Like waies the second mā had to his part 7128 li, the third must haue 6930 li. And the .iiij. man shall haue for his parte 13728 li, as these figures do parcelly declare

b

31950 — 35145
6480 — 7128

c

31950 — 35145
6300 — 6930

d

31950 — 35145
12480 — 13728

S. This I like very well: but what proofe is there of this worke? M. The same that I taught you for the other. Howbeit there is vsed both for this work & the other also this maner of proofe, to adde all the portiōs togither, & if thei agree to the hole sum, than semeth it wel done: but this is no sure rule. S. yet wil I proue it in this example. The four

Proofe

parcels ar these, which if I adde to- 7359
gither: there wil amoũt 35145, & 7128
that was the hole summe, so is this 6930
rule true here. M. And so will it be 13728
stil when the worke is truly done. But if you list to see it proued false, take 10000 li. from the fourth man, and put it to any of the other 3, & than be you sure that you haue not don wel, & yet wil that prose allowe it, for the addition will still be al one. S. It must nedes be so: but what haue I nowe to learne? M. There ar many other excellẽt partes behind of which I will not as now make mencion, bicause that without the knowlege of fractions, thei can not duely be taught, and much lesse vnderstand: Therfore will I propose to you. ii. or. iii. questions more, whereby you may practise the better the feate of the rule of felowship, and so make an end for this tyme. But this maie not be forgoten, that in all suche questions if the mony be of diuers kyndes, you must by reduction brynge it to one kynde, that is to saie, to the least valure that is named in the question. And like wais shall you doo, if the tyme be of diuers kyndes: as some yeres, some monethes, wekes, & daies: you shall make al monethes, wekes, or dais, accordyng as the least name of tyme in the question is: As for exãple. First in diuersitee of money: iii. cõpanions bought 2000 shepe & payd for them 241 poũd 13 s̃ 4 d̃, of which summe

Question of shepe.

summe one paid 101 li 10 ß. The secõd paid 82 li. 17 ß 10 ð. And the third paid 57 li. 5 ß 6 ð. How many shepe must eche of them haue? Answere: The first shal haue 840. The second 686. And the thyrde 474. And that must you worke thus: Fyrst considerynge that your mony is of dyuers denominatiõs, you shal (by reduction) bryng it all into the smallest denomination whiche is in it, that is to saie, pens, and so wil the total sum be 58000 ð. Nowe yf you tourne eche maus mony into pennies also, the fyrst mans summe will bee 24360 pens. The second mans summe 19894 ð. And the thyrde mannes money wyll bee 13746 ð. Nowe to knowe howe many sheepe euery man shall haue, sette the wholle summe of mony that is 58000 ð, in the fyrst place, and in the second place set the numbre of sheepe, and then orderly in the thyrde place sette eche mannes mony: and then multiplyng the thyrd and second summes togyther, and dyuydynge that that amounteth by the first, there will appeere the numbre of shepe that eche mã ought to haue as these thre figures doo shewe.

a

58000 — 2000
24360 — 840

b

58000 — 2000
19894 — 686

c

58000 — 2000
13746 — 474

Scolar. Why doo you sette the monye in the first place, seyng in the question you sat 2000 shepe coste 58000 ð, and not thus, 58000 ð cost 2000 shepe. M. And you remembre, I taught you at the begynnyng of this golden rule, that the fyrst and third numbre must be of one name, and of like thynges, and euermore the numbre that the question is asked of, must be set in the thyrd place. Now is the question plainely this. If 4 men boughte 2000 shepe for 58000 ð, howe many shepe shall eche man haue? But seyng in this question there ought more respect to be hadde to the summe of money, than to the summe of the persons (for in the summes of mony is there proporcion toward the shepe, and not in the numbre of persons) therfore must we turne the question, thus; If 58000 ð bought 2000 shepe, how many dyd 24360 ð bye? Agayne, howe many dyd 19894 ð bye? and how many bought 13746 ð? S. I perceaue it reasonable, and so shall I doo in all lyke questyons. M. Euen so. But for cause of the woorke, marke this: when so euer the fyrst & second nomber hath cyphers in theyr fyrst places, you may bothe in the multiplication and in the diuision leaue out those cyphers, so that you leue out lyke many out of bothe summes, as in this question the fyrste nomber 58000 hath thre cyphers, and so hathe the seconde, that is 2000,

ther-

therfore caste awaye theyr cyphers, and so will the fyrst numbre be 58, and the seconde 2 set them in theyr places, and worke accordyng to the rule, and you shall perceaue that it wil be al one, sauyng that this is the shorter and easier waie, as these thre figures do showe.

a

58 — 2
14360 — 840

b

58 — 2
19894 — 686

c

58 — 2
13746 — 474

And this you se is bothe the easier and also the more certaine waie, to know the answer to this question. S. Truthe it is as you saie: But syr, me semeth I might aske a farther question here, not onely howe many sheepe eche man shulde haue, but also what euery shepe coste. M. That question doeth not onely belonge to this rule, but maie also be discussed by Diuision, especially if the questions numbre be one onely: as thus. Diuide the totall summe 58000 d. by 2000 (other 58 by 2, omittyng the ciphers) and the quotient will be 29 d. that is 2 s̃, 5 d: howe be it, by this rule you maie doe it, and best, when the numbre of the question doeth excede 1: as if I shulde aske this question, 2000 shepe coste 52000 d. how muche dydde 20 coste? Then shall I set my figure thus.

2000

2000 ⟶ 58000 And doyng after the rule,
20 ⟶ there wil amount 580 d.
(that is 2 li. 6 s. 4 d.) the price of one score.
But if you wil vse that easy waie, that I did
teache you, you maie chaunge the firste and
seconde numbre, thus. 2 ⟶ 58
Yet now one question more will 20 ⟶
I moue (ẏ you maie perceaue ẏ vse of all o-
Question. ther lyke) & so make an end. There is in a ca-
thedrall churche 20 cannones, & 30 vycars,
those maie spend by yeare 2600 li. but euery
cannon muste haue to his parte 5 tymes so
muche as euery vicare hath: how muche is
euery mans portion saie you? S. I praie you
make the aunswere your selfe, so shal I per-
ceaue best the meanes to answere to suche o-
ther lyke. M. In this question you must doe
as in those that haue diuersitee of tyme, for
here is diuersitee of portions: Therfore shal
you multiply the numbre of the persons by
their difference of portion (as you did in the
other by tyme) then muste you multiply the
20 (which is the numbre of cannones) by
5 (for that is the nnmbre of their portion) so
will it be 100: then 30 (that is the numbre of
vicares) by 1 (that is the numbre of their
portion) and it will be 30: put those two ssi-
mes together, and thei make 130, then saie
thus: If 130 spende 2600 li. what maie 100
spende? The rule showeth 2000 li. Againe
for the vicares: If 130 spende 2600 pound,
what

what maie 30 spende? Answere: 600 li, as these figures showe.

130 — 2600 | 130 — 2600
100 — 2000 | 30 — 600

But if euery canone shulde haue so oftentymes 4 li, as the vicare shulde haue 3 li. then shoulde I multiply 20 by 4 (that were 80) and 30 by 3 (that were 90) and then bothe were 170. Then should the fygures bee sette thus.

li. s. d. | li. s. d.

170 — 2600 | 170 — 2600
80 — 1223, 10, 7 | 90 — 1376, 9, 5

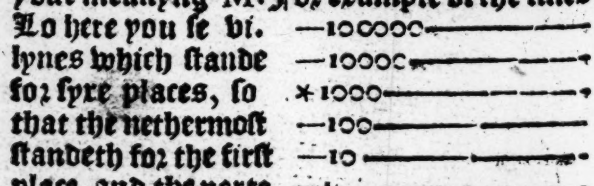
But this sorte is to harde for you by reason of the fractions, therfore I will let it rest to that place. And by this rule you se what the 20 cannones maie spende: whiche summe if you diuide by 20, you shall see eche cannons portion and so of the vicares, if you diuide they? summe by 30, the quotient wyl declare euery vicars portion.

The

The seconde dialogue of accoumpyng by counters.

Maister.

Owe that you haue learned the commen kyndes of Arithmetike with the penne, you shall see the same arte in coũters: whiche feate dooeth not onely serue for them that can not write & rede, but also for thẽ that can doe bothe, but haue not at some tymes their penne or tables ready with them. This sorte is in two fourmes cõmenly. The one by lynes, and the other withoutlynes: in that that hath lines, the lines do stand for the order of places: and in that that hath no lynes, there muste be sette in their stede so many counters as shall nede, for eche lyne one, and they shall supply the stede of the lynes. S. By examples I shuld better perceaue your meanyng. M. For example of the lines Lo here you se vi. lynes which stande for syre places, so that the nethermost standeth for the first place, and the nexte



aboue

aboue it, for the seconde: and so vpwarde til you come to the hyghest, whiche is the fyrste lyne, and standeth for the fyrste place. Now what is the valewe of euery place or lyne, you maie perceaue by the figures whiche I haue set on them, which is accordyng as you learned before in the Numeratiõ of figures by the penne: for the fyrst place is the place of vnites or ones, and euery counter set in that lyne betokeneth but one: and the second lyne in the place of 10, for euery coũter there standeth for 10. The thyrde lyne the place of hundredes: the fourth of thousandes: and so forth. S. Syr I doe perceaue that the same order is here of lynes, as was in the other figures by places, so that you shall not nede longar to stande about Numeration, except there be any other difference. M. If you doe vnderstande it, then how will you set 1543? S. Thus, as I suppose. M. You haue set the places truely, but your figures be not mete for this vse: for the meteſt figure in this behalfe, is the figure of a counter rounde, as you se here, where I haue expressed that same summe. S. So that you haue not one figure for 2, nor 3, nor 4, and so forth, but as many digettes as you haue, so many counters you sette in the

lowest

The seconde dialogue of accumpsyng by counters.

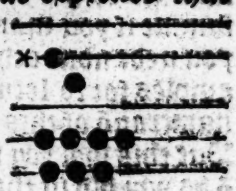
Maister.

Owe that you haue learned the commen kyndes of Arithmetike with the penne, you shall see the same arte in coũters: whiche feate dooeth not onely serue for them that can not write & rede, but also for thẽ that can doe bothe, but haue not at some tymes their penne or tables ready with them. This write is in two fourmes cõmenly. The one by lynes, and the other without lynes: in that that hath lines, the lines do stand for the order of places: and in that that hath no lynes, there muste be sette in their stede so many counters as shall nede, for eche lyne one, and they shall supply the stede of the lynes. S. By examples I shuld better perceaue your meanyng. M. For example of the lines Lo here you se vi. lynes which stande for syxe places, so that the nethermost standeth for the first place, and the nexte

—100000—
—10000—
—1000—
—100—
—10—
—1—

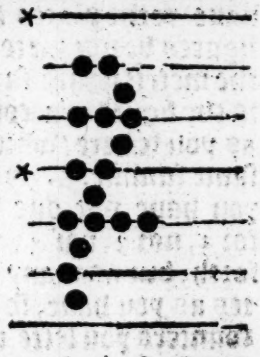
aboue

aboue it, for the seconde: and so vpwarde til you come to the hyghest, whiche is the fyrste lyne, and standeth for the fyrste place. Now what is the valewe of euery place or lyne, you may perceaue by the figures whiche I haue set on them, which is accordyng as you learned before in the Numeratiõ of figures by the penne: for the fyrst place is the place of vnites or ones, and euery counter set in that lyne betokeneth but one: and the second lyne in the place of 10, for euery counter there standeth for 10. The thyrde lyne the place of hundredes: the fourth of thousandes: and so forth. S. Syr I doe perceaue that the same order is here of lynes, as was in the other figures by places, so that you shall not nede longar to stande about Numeration, except there be any other difference. M. If you doe vnderstande it, then how will you set 1543? S. Thus, as I suppose. M. You haue set the places truely, but your figures be not mete for this vse: for the meetest figure in this behalfe, is the figure of a counter rounde, as you se here, where I haue expressed that same summe. S. So that you haue not one figure for 2, nor 3, nor 4, and so forth, but as many digettes as you haue, so many counters you sette in the



lowest

lowest lyne: and for euery 10 you set one in the seconde lyne: & so of other. But I know not by what reason you set that one counter for 500 betwene two lynes. M. You shal remembre this, that when so euer you nede to set downe 5, 50, or 500, or 5000, or so forth any other numbre, whose numerator is 5, you shall set one counter for it in the nexte space aboue the lyne that it hath his denomination of, as in this example of that 500, bycause the numeratour is 5, it muste be set in a voide space: and bycause the denominator is hundred, I know that his place is the voide space nexte aboue hundredes, that is to saie, aboue the thyrd lyne. And farther you shall marke, that in all workyng by this sorte, if you shall sette downe any summe betwene 4 and 10, for the firste parte of that numbre, you shal set downe 5, & then so many counters more, as there reste numbres aboue 5. And this is true bothe of digettes & articles. And for example I will sette downe this summe 287965, which summe if you marke wel you nede none other examples for to lerne the numeratiō of this form But this shal you marke ẏ as you did in the other

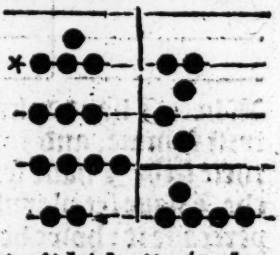


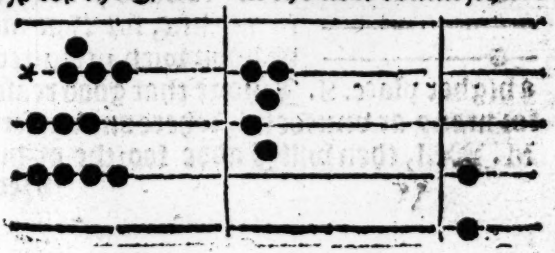
kynd of arithmetike, set a pricke in ẏ places

of

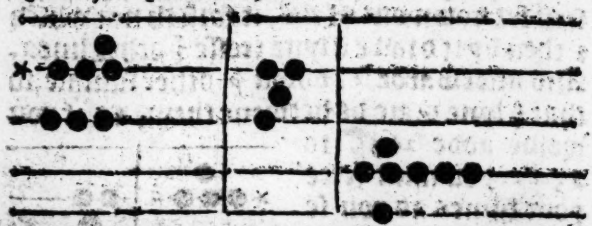
of thousãdes, in this worke you shal set a stare as you se here. S. Then I perceaue numeration: but I praie you, how shal I do in this arte to adde two summes or more together?

M The easiest waie in this, arte is to adde but 2 sũmes at ones together: how be, it you may adde more as I wil tel you anone, therfore when you wil adde two sũmes, you shal first set down one of thẽ, it forseth not which & then by it drawe a lyne crosse ỹ other lines. And afterwarde set down ỹ other summe, so that ỹ lyne maie be betwene them, as if you wolde adde 2659 to 8342, you must sette your sũmes as you se here. And then if you lyst, you mai adde the one to the other in the same place, or els you maie adde them both together in a newe place: whiche waie, bycause it is most plainest, I wil show you first Therfore wil I beginne at ỹ vnitees, which in the firste summe is but 2, & in the seconde summe 9, that maketh 11, those doe I take vp, & for thẽ I set 11 in the new roume, thus. Addition.

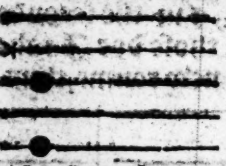




Then doo I take vp all the articles vnder a hundred, whiche in the fyrst summe are 40, and in the seconde summe 50, that maketh 90: or you maie saie better, that in the first summe there are 4 articles of 10, and in the seconde summe 5, whiche make 9, but then take hede, that you set theim in theyr ryght lynes, as you see here.



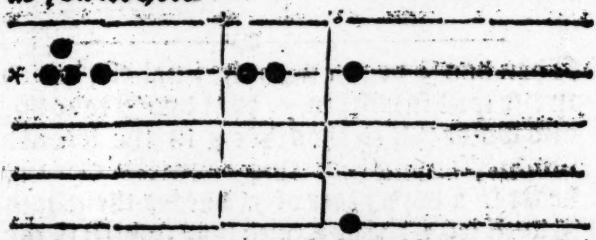
Where I haue taken awaye 40 frome the fyrst summe, and 50 from the second, and in their stede I haue set 90 in the thyrde, whiche I haue set playnly, that you might well perceaue it: howe be it seynge that 90 with the 10 that was in the thyrd roume al redy, dooth make 100, I might better for those 6 counters sette 1 in the thyrde lyne, thus:



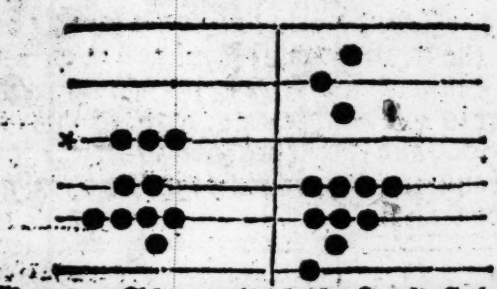
For it is al one summe as you maie see, but it is best neuer to sette 5 counters in any line, for that maie be doon with 1 counter in a higher place. S. I iudge that good reason, for many ar vnnedeful, where one wil serue. M. Well, then will I adde foorthe of hun-

dredes

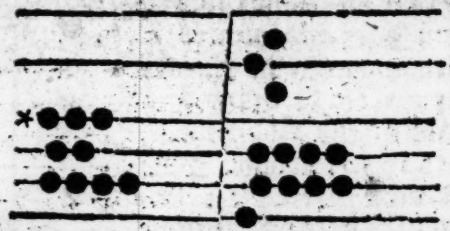
dredes: I fynde 3 in the fyrste summe and 6 in the seconde, whiche make 9000, then doo I take vp and sette in the thyrde roume where is one hundred al readye, to which I put 900, and it will bee 1000, therfore I set one counter in the fourthe lyne for them all, as you see here.



Then adde I the thousandes togither, whiche in the fyrste summe are 8000, and in the seconde 2000, that maketh 10000: theim doo I take vp frome those two places, and for them I set one counter in the fyfte lyne, and then appereth as you see to be 11001, for so many wth amounte of the Addition of 8342 to 2659. Scolar. Syr, this I dooe perceaue: but howe shall I sette one summe to an other, not chaungynge theym to a thyrde place? Mayster. Marke well howe I doe it: I wylle adde togyther 65436, and 3245, whyche fyrst I sette downe thus.



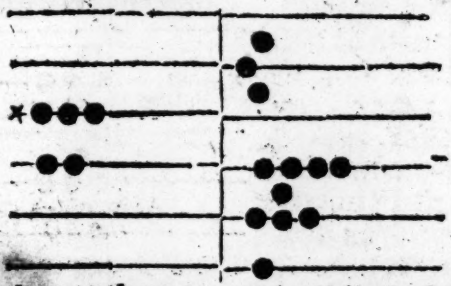
Then doo I begyn with the ſmalleſt, which in the fyrſt ſumme is 5, that doo I take vp, and wolde put to the other 5 in the ſeconde ſumme, ſauyng that two counters can not be ſet in a voyd place of 5, but for them both I muſt ſet 1 in the ſecond lyne, which is the place of 10, therfore I take vp the 5 of the fyrſt ſum and the 5 of the ſecond, & for them I ſet 1 in the ſeconde lyne, as you ſe here.



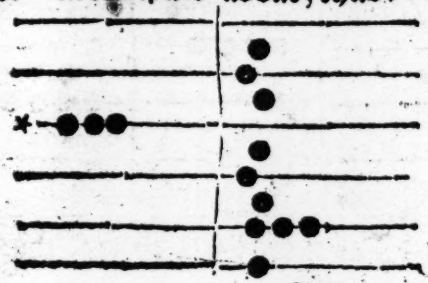
Then doo I lyke wais take vp the 4 counters of the fyrſte ſumme and ſeconde lyne, (whyche make 40) and adde theim to the 4 counters of the ſame lyne in the ſeconde ſumme, and it maketh 80. But (as I ſayde) I maie not conueniently ſet aboue 4 counters

ters in one lyne, therefore to those 4 that I tooke vp in the fyrst summe, I take one also of the seconde summe, and then haue I taken vp 50, for whiche 5 counters I sette downe one in the space ouer the second lyne, as here dothe appere.

73
41
62

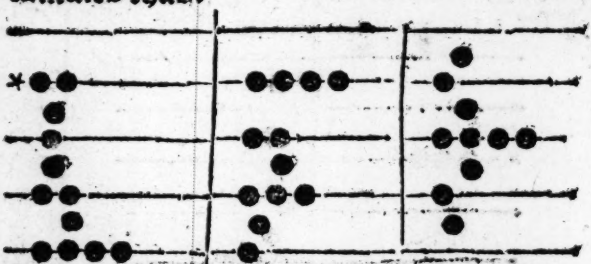


And than is there 80, as well with those 4 counters as yt I had sette downe the other 4 also. Nowe doe I take the 200 in the firste summe, and adde theim to the 400 in the second summe, and it maketh 600, therfore I take vp the 2 coũters in the fyrst sum, & 3 of them in the second sume, and for them I set 1 in the space aboue, thus.

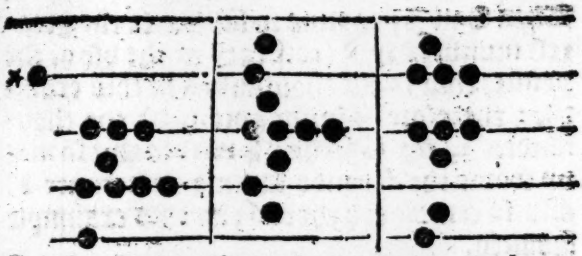


Then I take the 3000 in the fyrſt ſumme vn to which there ar none in the ſecond ſūme, a greyng, therfore I doo onely remoue thoſe 3 counters from the fyrſt ſumme into the ſeconde, as here doothe appere.

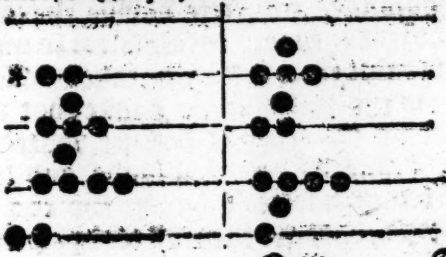
And ſo you ſe the holle ſumme that amounteth of the additiō of 65436 with 3245, to be 68681. And if you haue marked theſe two exaumples wel, you nede no farther inſtruction in addition of 2 only ſummes: but if you haue more then two ſummes to adde, you maye adde theim thus. Fyrſt adde two of them, and then adde the thyrde and the fourth, or more, if there bee ſo many: as if I woulde adde 2679 with 4286 and 1391. Fyrſt I adde the two fyrſt ſummes thus.



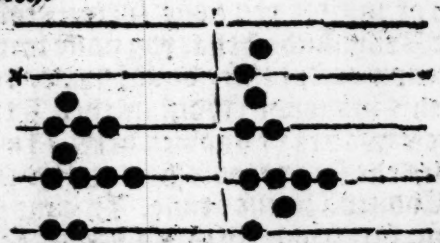
And then I adde the thyrd therto, thus.



And so of more if you haue them. Scolar. Nowe I thynke best that you passe forth to subtraction, except there be any waies to examyn this maner of Addition, then I think that were good to be knowen next. Maister. There is the same proofe here that is in the other Addition by the penne. I meane Subtraction, for that onely is a sure waie: but consyderyng that Subtraction must be first knowen, I wyll fyrste teache you the arte of Subtraction, and that by this example: I woulde subtracte 2892 oute of 8746. These summes must I set downe, as I dyd in Addition: but here it is best to set the lesser numbre fyrst, thus. Subtraction.

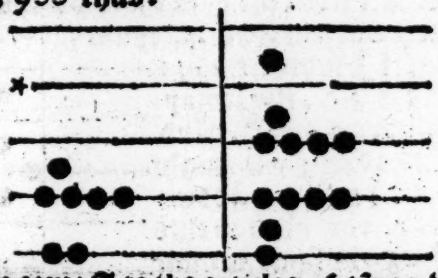


Then shall I begynne to subtracte the greatest numbres fyrst (contrary to the vse of the penne) that is the thousandes in this example: therefore I fynde amongest the thousandes 2, for whiche I withdrawe so many frome the seconde summe (where are 8) and so remaineth there 6, as this exaumple showeth.



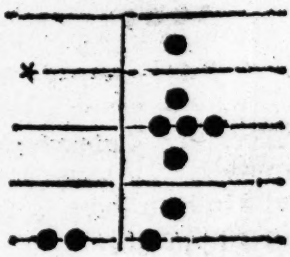
Than do I lyke waies with the hundredes, of whyche in the fyrste summe I fynde 8, and in the second summe but 7, out of whyche I can not take 8, therfore this muste I doo: I muste looke howe muche my summe differeth from 10, whiche I fynde here to be 2, then must I bate for my summe of 800 one thousande, and sette downe the excesse of hundredes, that is to saie 2, for so muche 1000 is more then I shoulde take vp. Therfore from the fyrste summe I take that 800, and frome the seconde summe where are 6000, I take vp one thousande, and leaue 5000: but then sette I downe the 200 vnto the 700, that are there all ready, & make them

them 900 thus.

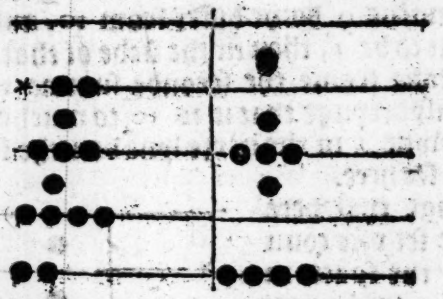


Then come I to the articles of tēnes, where in the fyrst ſumme I fynde 90, and in the ſeconde ſumme but onely 40: Now conſyderyng that 90 can not be bated frome 40, I looke howe muche that 90 doothe differ from the next ſumme aboue it, that is 100, or (elles whiche is all to one effecte) I looke howe muche 9 dothe differ from 10, and I fynde it to be 1, then in the ſtede of that 90, I doo take frome the ſeconde ſumme 100: but conſyderynge that it is 10 to muche, I ſette downe 1 in the nexte lyne benethe for it as you ſee here.

Sauynge that here I haue ſet one counter in the ſpace in ſtede of 5 in the next lyne. And thus haue I ſubtracted all ſaue twoo, whyche

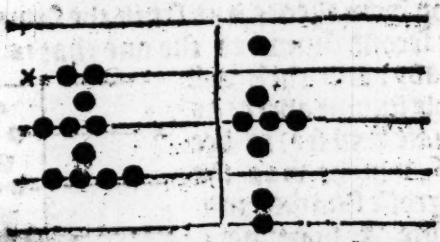


I muſt bate from the 6 in the ſecõd ſumme, and there wil remayne 4, thus. So that if I ſubtract 2892 frome 8746, the remayner will bee 5854. And that this is truely wrought you maye proue by Addition, for yf you adde to this remayner the ſame ſumme that you dyd ſubtract, than will the former ſumme 8746 amounte agayne. Scolar. That will I proue: and firſt I ſet the ſumme that was ſubtracted, whiche was 2892, and than the remainer 5854, thus,

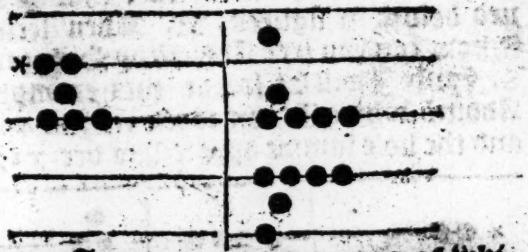


Then doo I adde fyrſte the 2 to 4, whyche maketh 6, ſo take I vp 5 of thoſe counters, and in their ſtede I ſette 1 in the ſpace, as here appeareth.

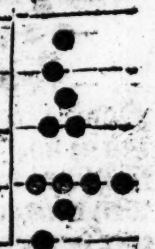
Then



Then doe I adde the 90 nexte aboue to the 50, and it maketh 140, therfore I take vp those 6 counters, & for the I set 1 to the hundredes in the thyrd lyne, and 4 in the second lyne, thus.

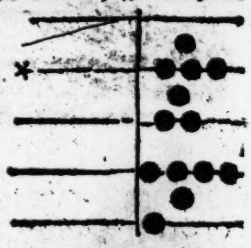


Then doe I come to the hundredes, of whiche I fynde 8 in ye fyrst summe, and 9 in the secõd, that maketh 1700, therfore I take vp those 9 counters, & in their stede I sette 1 in the iiij. lyne, and 1 in the space next benethe, and 2 in the thyrde lyne, as you see here.

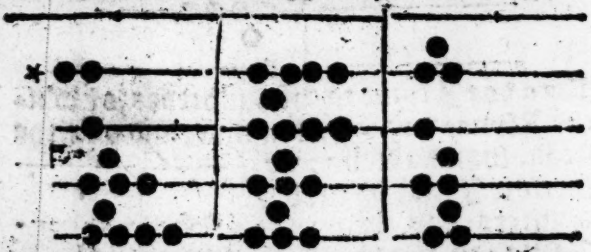


Then is there left in the fyrste summe but onely 2000, whiche I shall

take vp from thence, and set in the same lyne in the second summe to the one that is there all ready: and then will the hole summe apere (as you maie well see) to bee 8746, whiche was the firste grosse summe, and therefore I dooe perceaue that I hadde well subtracted before.

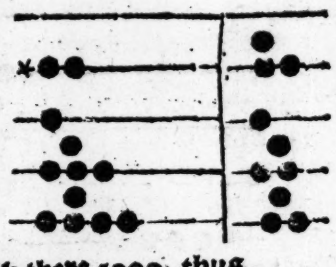


And thus you maie se how subtraction maie be tried by addition. S. I perceaue the same order here with counters, that I learned before in figures. M. Then lette me se how can you trye Additiō by Subtractiō. S. Fyrste I will set forthe this example of Additiō, where I haue added 2189 to 4988, and the hole summe appereth to bee 7177.



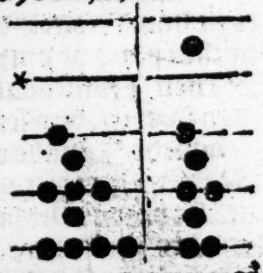
Now to trye whether that summ be well added or no, I will subtracte one of the fyrste two summes frome the thyrd, and if I haue well done, the remainer will be lyke that other summe, as for example: I wil subtracte the

the fyrſte ſumme from the thyrde, whiche I ſet thus in theyr order. Then doe I ſubtracte 2000 of the fyrſt ſumme from the ſeconde ſume,



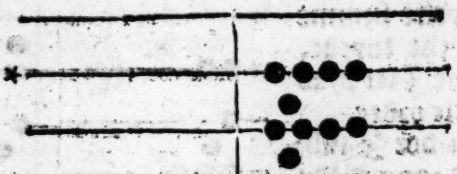
and then remaineth there 5000, thus.

Then in the thyrd lyne, I ſubtracte the 100 of the fyrſte ſumme frome the ſecond ſume where is onely 100 alſo, and then in the thyrde lyne reſteth nothyng. Then in the ſeconde lyne with

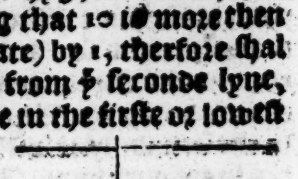


his ſpace ouer hym, I fynde 80, whiche I ſhoulde ſubtracte from the other ſumme, then ſeyng there are but onely 70, I muſt take it out of ſome higher ſumme, whiche is here onely 5000, therefore I take vp 5000, and ſeynge that it is to muche by 4920, I ſet downe ſo many in the ſeconde roume, whiche with the 70 beyng there all ready doe, make 4990, and then the ſummes doeth ſtande thus.

Yet



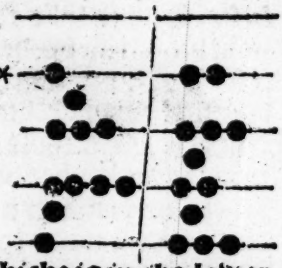
Yet remaineth there in the firste ſumme 9 to be bated from the ſeconde ſumme, where in that place of vnitees dothe appere onely 7: then I muſte bate a hygher ſumme, that is to ſaie 10, but ſeyng that 10 is more then 9 (whyche I ſhulde abate) by 1, therfore ſhal I take vp one counter from ỹ ſeconde lyne, and ſet downe the ſame in the firſte or lowest lyne, as you ſe here.



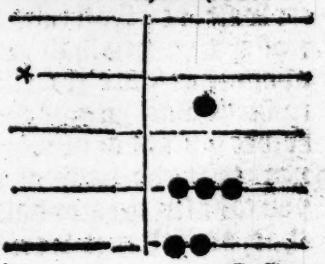
And ſo haue I ended this worke, and the ſumme appereth to be the ſame, which was the ſeconde ſumme of my addition, & therfore I perceaue, I haue well done. M. To ſtande longar about this it is but folye, excepte that this you maie alſo vnderſtande, that many do begyn to ſubtract with counters, not at the higheſt ſumme, as I haue taught you, but at the nethermoſte, as thei doe vſe to adde: & when the ſumme to be abated in any line appereth

greater

greater then the other then doe thei borowe one of the next higher roume, as for example: if they shoulde abate 1846 from 2378 thei sette the summes thus.



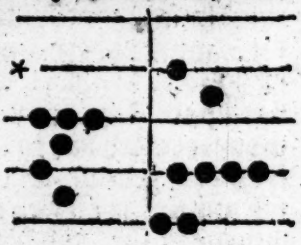
And fyrst thei take 6 whiche is in the lower lyne, and his space from 8 in the same roumes, in the seconde sum, and yet there remaineth 2 counters in the lowest lyne. Then in the second lyne must 4 be subtract frō 7, & so remaineth there 3. Then 2 in ỹ thyrde lyne & his space frō 3 of ỹ secōd sum cā not be, therfore do thei bate it from a hygher rome, ỹ is, from 1000, & because that 1000 is to much by 200, therfore must I set down 200 in the third lyne, after I haue taken vp 1000, from the fourth lyne, then is there yet 1000 in the fourth lyne of the firste summe, whiche if I withdrawe from the seconde summe, then doeth all the figures stande in this order.



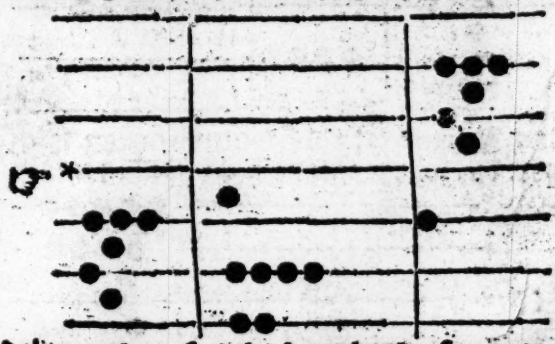
So that (as you se) it differeth not gretly whether you begynne subtraction at the higher lynes or at ỹ lower. Howbeit, as some men lyke the one waie beste,

beste, so some lyke the other: therfore you now knowyng bothe, maie vse whiche you lyst. But now touchynge Multiplication: you shal set your numbres in two roumes (as you did in those two other kides) but so that the multiplier be set in the firste roume. Then shal you begyn with the highest numbres of the seconde roume, and multiply the fyrste after this sorte. Take that ouermoste lyne in your fyrste worckyng, as if it were the lowest lyne, settig on it some mouable marke (as you lyste), & loke how many counters be in hym, take them vp, and for the set downe the hole multiplier, so many tymes as you toke vp counters (reckenyng) I sate: ♀ lyne for the vnites: and when you haue so done with the highest numbre, the come to ♀ nert lyne beneth, & do euen so with it, and so with the nert till you haue done all. And if there be any numbre in a space, the for it shall you take the multiplier 5 tymes, and then muste you recken that line for the vnitees which is nerte benethe that space: or els after a shorter waie, you shall take onely halfe the multiplier, but then shall you take the lyne nerte aboue that space for the lyne of vnitees: but in suche worckyng, if chaunce your multiplier bee an odde numbre, so that you can not take the halfe of it iustely, then muste you take the greater halfe, and sette downe that, as if that it were the iuste halfe, and far-

farther you shall set one counter in the space beneth that lyne, whiche you recken for the lyne of vnitees, or els onely remoue forward the same that is to be multiplied. S. If you set forth an example hereto, I thynke I shal perceue you. M. Take this exāple: I wold multiply 1542 by 365, therefore I set the numbres thus.



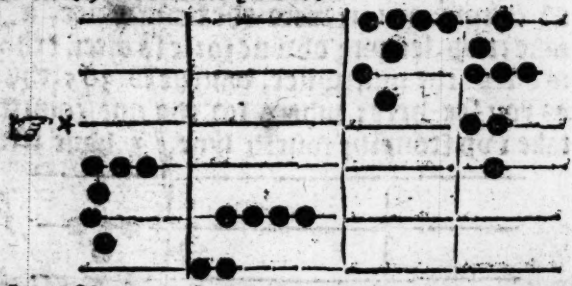
Then first I begyn at the 1000 in the highest roume: as if it were the firste place, & I take it vp, settyng downe for it so often (that is ones) the multiplier, whiche is 365, thus as you see here: where for the one counter taken vp from the fourth lyne, I haue sette



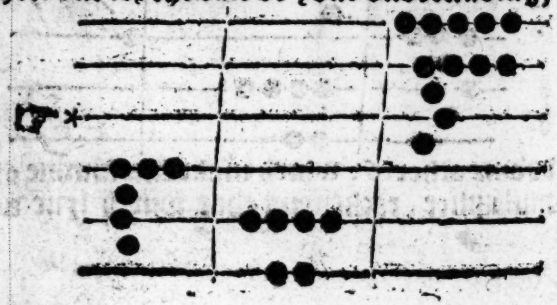
downe other 6, which make the summe of ye multiplier, reckenyng that fourth lyne as if

it

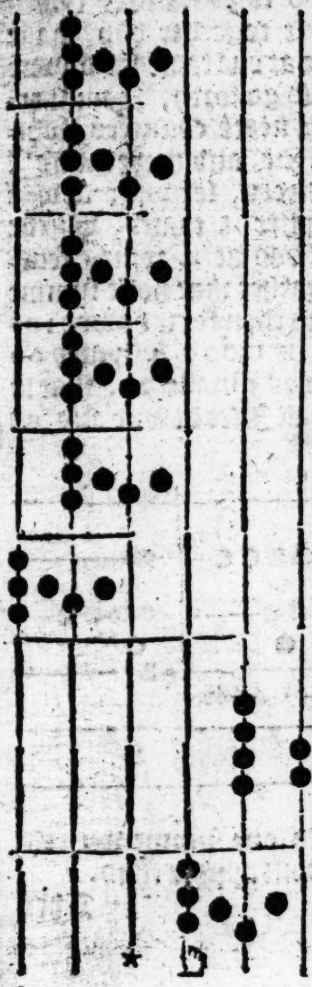
it were the fyrste, which thyng I haue marked by the hande ſet the begynnynge of the ſame, S. I perceaue this well, for in dede, this ſume that you haue ſet down is 365000 for ſo muche doeth amount of 1000, multiplied by 365. M. Wel, then to go forth in the next ſpace I fynde one counter, which I remoue forward but take not vp, (but do as in ſuch caſe I muſt) ſet down ye greater halfe of my multiplier (ſeyng it is an odde numbre) whiche is 182, and here I doe ſtyll lette that fourthe place ſtande, as if it were the fyrſte: as in this fourme you ſee.



where I haue ſet this multiplication with other: but for the eaſe of your vnderſtãding,

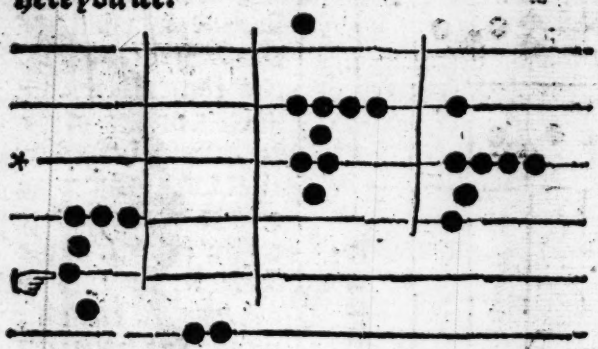


I haue set a litteli line betwene them: nowe shulde thei bothe in one summe stande thus.



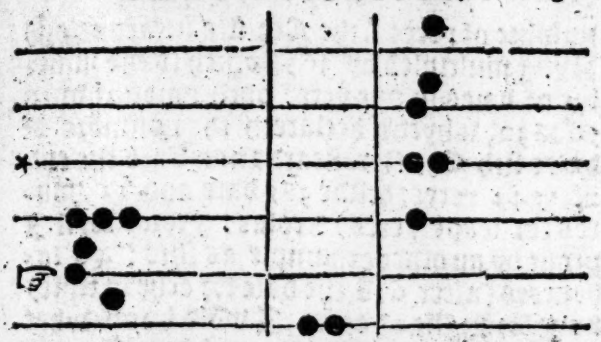
How be it an other fourme to multiply suche counters in space is this: Fyrst to remoue the finger to the lyne nexte beneth that space, and then to take vp that counter, and to sette down the multiplier v. tymes, as here ou se. Which summes if you do adde togyther into one summe, you shal perceaue that it will be the same that appereth of the other woorkyng before, so that bothe sortes ar to one entente, but as the other is more shorter, so this is playner to reason, for such as haue had smal exercise in this arte. Notwithstãdyng you make adde them in your mynde

fore you sette them downe, as in this example you myght haue saide, 5 tymes 300 is 1500, and 5 tymes 60 is 300: also 5 tymes 5 is 25, whiche all put together doo make 1825, which you maie at one tyme set downe if you lyst. But now to go forth, I must remoue the hande to the nexte counters, whiche are in the seconde lyne, and there muste I take vp those 4 counters, settynge downe for theim my multiplier 4 tymes, whiche thynge other I maye doo at 4 tymes seuerally, or els I maie gather that holle summe in my mynde fyrst, and then set it downe: as to saie, 4 tymes 300 is 1200: 4 tymes 60 are 240: and 4 tymes 5 make 20, that is in all 1460, that shall I set downe also, as here you see.

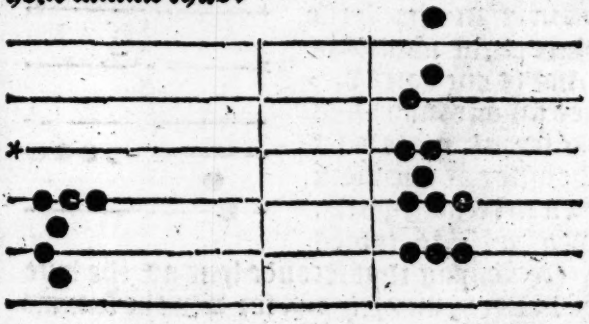


Whiche yf I ioyne in one summe with the former numbres, it will appere thus.

Then



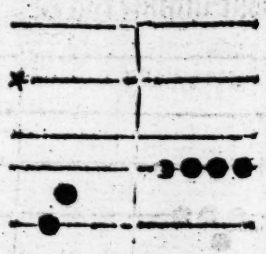
Then to end this multiplication, I remoue the fynger to the lowest lyne, where are only 2, them doo I take vp, and in their steede do I sette downe twise 365, that is 730, for whiche I set one in the space aboue the third lyne for 500, and 2 more in the thyrde lyne with that one that is there all redy, and the reste in their order, and so haue I ended the holle summe thus.



Wherby you see, that 1542 (whiche is the

Daies syth Christis incarnation.

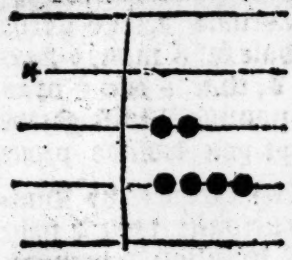
numbze of yeres sithe Chzistis incarnation) beyng multipled by 365 (which is the numbze of daies in one yere) doth amount vnto 562830, whyche declareth the noumbze of daies sith Chzistis incarnation vnto the end of 1542 yeres (beside 385 dais and 12 houres foz leape yeres) Scolar. Nowe wyll I pzoue by an other exaumple, as this: 40 labourers (after 6 d the daie foz eche manne) haue wzought 28 daies, I wold know what their wages doothe amounte vnto: In this case muste I woozke doublely: fyzst I must multiplie the numbze of the laboters by the wages of a manne foz one daie, so will the charge of one daie amount: then secondarily shall I multiplie that charge of one daie, by the holle numbze of daies, and so will the holle summe appere: fyzst therfoze I shal set the summes thus.



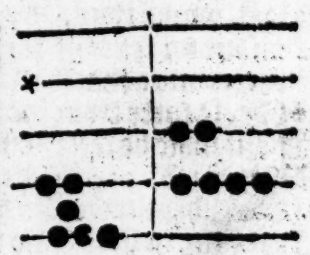
Where in the fyrste place is the multiplier (that is one daies wages foz one man) & in the second space is set the nũbze of the wozkmen to be multiplied: then saye I, 6 tymes 4 (reckenyng that seconde lyne as the lyne of vnites) maketh 24, foz whyche summe I should sette 2 counters in the thyzd lyne, and 4 in the seconde, therfoze doo I sette 2

in

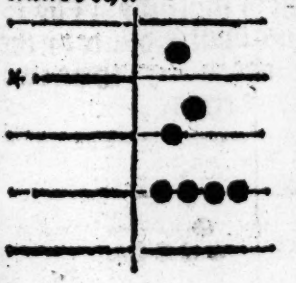
in the thirde lyne, and lette the 4 ſtand ſtill in the ſeconde lyne, thus.



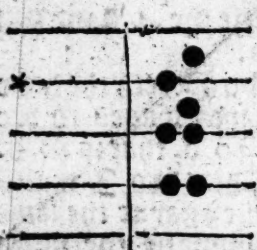
So apereth the holle dayes wages to bee 240 d, that is 20 s. Then doo I multiplie agayne the ſame ſumme by the numbze of daies, and fyrſt I ſette the numbzes, thus.



Then bycauſe there ar counters in diuers lynes, I ſhall begyn with the higheſt and take theym vp, ſettynge for theym the multiplyer ſo many tymes as I tooke vp counters, that is twiſe, then will the ſumme ſtande thus.



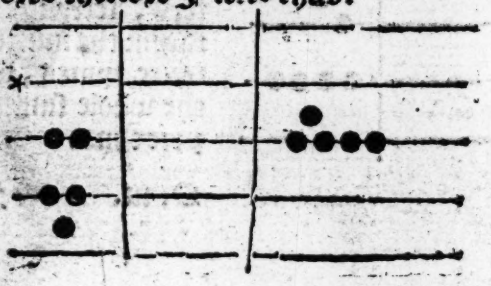
Then come I to the ſeconde lyne, and take vp thoſe 4 counters, ſettynge for theym the multiplier foure tymes, ſo will the whole ſumme appere thus.



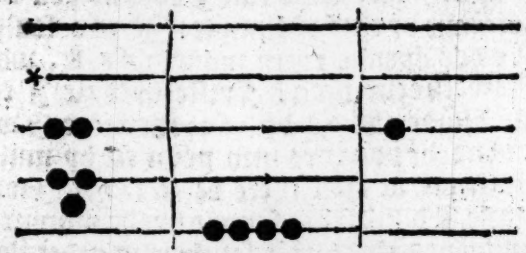
So is the hole wages of 40 woorkemen for 28 dais (after 6 d eche daie for a man) 6720 d, that is 560 s, or 28 pounde. Maist. Nowe yf you woulde proue Multiplication, the surest waie is by Diuision: therfore will I ouerpasse it tyll I haue taught you the arte of Diuision, which you shall woorke thus. Fyrst sette downe the Diuisour for feare of forgettyng, and than sette the numbre that shall be diuided, at the right side, so farre from the diuisour, that the quotient maie be sette betwene them: as for example: If 225 shepe cost 45 li, what dydde euery shepe cost? To knowe this, I shuld diuide the holle summe that is 45 li by 225, but that can not be, therfore must I fyrst reduce that 45 li into a lesser denomination, as into shillynges: the I multiply 45 by 20 & it is 900, that summe shall I diuide by the numbre of shepe, whiche is 225, these two numbres therfore I sette thus.

Diuision.

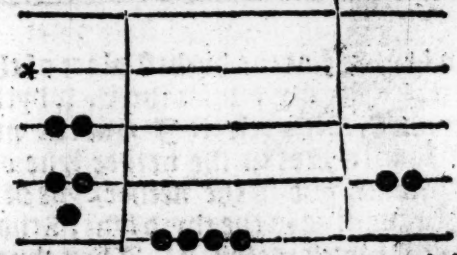
Exãple of shepe.



Than begyn I at the highest lyne of the diuident, and seke how often I maie haue the diuisour therin, and that maie I doo 4 tymes, then saie I: 4 tymes 2 are 8, whiche if I take from 9, there resteth but 1, thus.



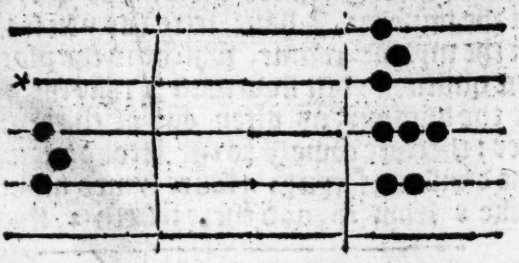
And bycause I founde the diuisor 4 tymes in the diuident, I haue sette (as you see) 4 in the myddle roume, whiche is the place of the quotient: but now must I take the reste of the diuisour, as often out of the remayner: therfore come I to the seconde lyne of the diuisour, saiyng: 2 foure tymes make 8, take 8 from 10, and there resteth 2, thus.



Then come I to the lowest numbre, whiche is 5, and multiplie it 4 tymes, so is it 20,

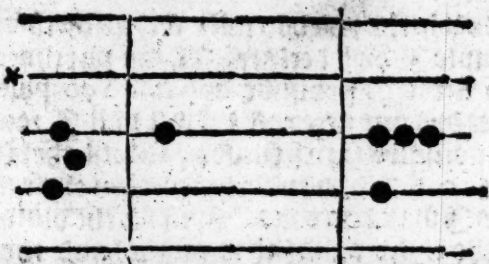
that take J from 20, and there remayneth nothyng, so that J set my quotient to be 4, whyche are in balewe shillyuges, for so was the diuident: and thereby J knowe, that yt 225 shepe dyd coste 45 li, euery shepe coste 4 s. Scholar. This can J doo, as you shall perceaue by this exaumple: If 160 souldiors doo spende euery moneth 68 li, what spendeth eche man? Fyrste byeause J can not diuide the 68 by 160, therfore J will tourne the poundes into peunies by multiplication, so shall there be 16320 li. Nowe muste J diuide this summe by the numbre of souldiours, therfore J set them in order thus

Exrample of souldiours wages.

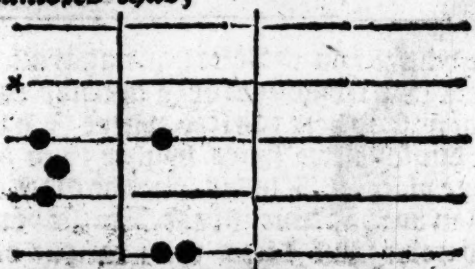


Then begyn J at the highest place of the diuident, seking my diuisor there, whyche J fynde ones, therfore sette J 1 in the nether lyne. Maist. Not in the nether lyne of the holle summe, but in the nether lyne of that woorke, whyche is the thyrd lyne. Scho. So standeth it with reason. M. Then thus doo they stande.

Then



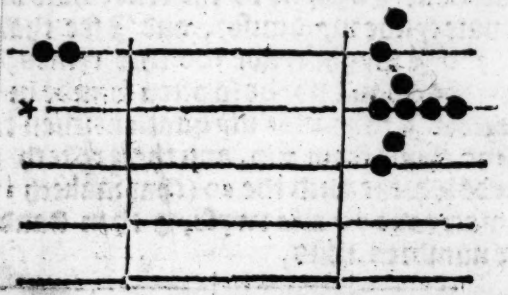
Then seke I agayne in the reste, how often I maie fynde my diuisor, and I see that in the 300 I myght fynde 100 thre tymes, but then the 60 wil not be so often founde in 20, therfore I take 2 for my quotient: then take I 100 twyse from 300, and there resteth 100 out of whiche with the 20 (that maketh 120) I maie take 60 also twyse, & then standeth the numbres thus,



where I haue sette the quotient 2, in the lowest lyne: So is euery sowldiours portion 102 d, that is. 8 ß 6 d. Mayster. But yet because you shall perceaue iustely the reason of Diuision, it shall be good that you do set your diuisour stille againste those numbres

Exʒple of purchase.

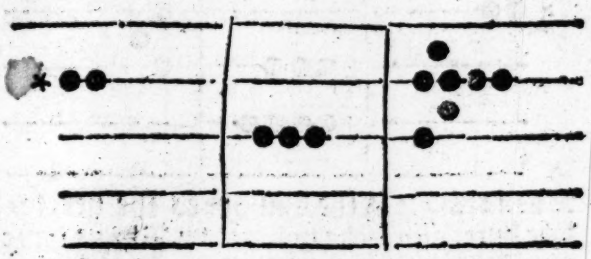
from whiche you doo take it, as by this exaumple I wyll declare. If the purchace of 200 acres of grounde dyd coſt 290 pound, what dyd one acre coſt? Fyꝛſt will I tourne the poundes into pennies, ſo will there bee 69600 d. Then in ſettyng down theſe numbꝛes I ſhall doo thus. Fyꝛſt ſet the diuident on the right hand (as it ought) and then



the diuiſour on the lefte hand agaynſt thoſe numbꝛes, frome whiche I entende to take hym fyꝛſt as here you ſee, where I haue ſet the diuiſour two lynes hygher than is his owne place. S. This is lyke the oꝛder of diuiſion by the penne. Maiſt. Truthe you ſaie, and nowe muſt I ſette the quotient of this wooꝛke in the thyꝛd lyne, foꝛ that is the lyne of vnitees in reſperte to the diuiſour in this wooꝛke. Then I ſeeke how often the diuiſoꝛ maie be founde in the diuident, and that I fynde 3 tymes, then ſette I 3 in the thyꝛde lyne foꝛ the quotient, and take awaye that

60000

60000 frome the diuidente, and farther I doe sette the diuisour one lyne lower, as you see here.



And then seeke I howe often the diuysour will be taken frome the numbre agaynste it, whiche will be 4 tymes, and 1 remaynyng. S. But what if it chaunce that when the diuisour is so remoued, it can not bee ones taken out of the diuident agaynst it? Maister. Then must the diuisour bee sette in an other lyne lower. Scoler. So was it in diuision by the penne, and therfore was there a cypher sette in the quotient: but howe shall that be noted here? M. Here nedeth no token, for the lynes do represent the places: onely loke that you set your quotient in that place, whi che standeth for vnitees in respect of the diuisour: but now to retourne to the example, I fynd the diuisor 4 tymes in the diuident, & 1 remaynyng, for 4 tymes 2 make 8, which I take from 9, and there resteth 1, as this figure sheweth: and in the myddle space for

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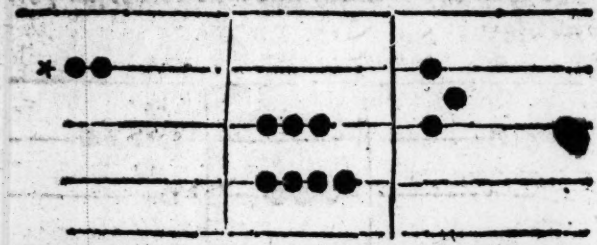
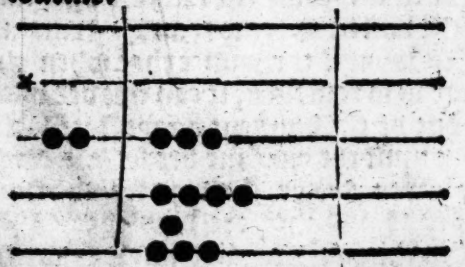
the quotient I sette 4 in the seconde lyne, which is in this worke the place of vnitees.



Then remoue I the diuisour to the next lower lyne, and seeke howe often I maie haue it in the diuident, whiche I maie doo here 8 tymes iuste, and nothynge remayne, as in this fourme.



Where you maie see that the holle quotient is 348 ℔, that is 29 ℈, whereby I knowe that so muche coste the purchase of one acre. &c. Nowe resteth the prooues of Multiplication, and also of Diuision. M. There best prooues are eche one by the other, for Multiplication is prooued by Diuision, and Diuision by Multiplication, as in the woorke by

the

the penne you learned. Scolar. If that be all, you shall not nede to repete agayne that, that was sufficiently taughte all ready: and excepte you will teache me any other feate, here maye you make an ende of this arte, I suppose. Mai. So will I doo as touchynge wholle numbre, and as for broken numbre, I will not trouble your wit with it, tyll you haue practised this so well, that you be full perfect, so that you nede not to doubt in any poynt that I haue taught you, and the may I boldly instruct you in the art of fractions or broken numbre, wherin I will also shewe you the reasons of all that you haue nowe learned. But yet before I make an ende, I will shewe you the order of common castyng, wherein are bothe pennies, shillynges, and poundes, procedyng by no grounded reason, but onely by a receaued fourme, and that diuersely of diuers men: for marchantes vse one fourme, and auditours an other: But fyrst for marchauntes forme, marke this example here, in whyche I haue expressed this summe 198 li, 19 s 11 d. So that you may see, that the lowest lyne serueth for pennies, the nexte aboue for shyllynges, the thyrde for poundes, and the fourthe for scores of poundes. And farther you may se that the space betwene pens and shyllynges may

maie receaue but one counter (as all other spaces lyke wayes doo) and that one standeth in that place for 6 d. Lyke waies betwene the shyllynges and the poundes, one counter standeth for 10 s. And betwene the poundes and 20 l. one counter standeth for 10 poundes. But beside those you maie see at the left syde of shyllynges, that one counter standeth alone, & betokeneth 5 s. So against ꝑ poundes, ꝑ one counter stãdeth for 5 li. And against ꝑ 20 poundes, the one coũter standeth for 5 score poundes, ꝑ is 100 li, so ꝑ euery syde coũter is 5 tymes so much as one of thẽ against which he standeth. Now for ꝑ accoumpt of auditors take this exãple.

Where I haue expressed the same sũme 198 li. 19 s. 11 d. But here you see the pennies stande towarde the right hande, and the other encreasynge orderly towarde the lefte hande. Againe you maie see, that auditors will make 2 lynes (yea and more) for pennies, shillynges, and all other valewes, if their summes extende therto. Also you see, that thei set one counter at the right ende of eche rowe, which so set, there standeth for 5 of that roume: and on the lefte corner of the

rowe

rowe it standeth for 10 of the same rowe. But now if you wolde adde other subtracte after any of bothe those sortes, if you marke the order of the other feate which I taught you you maie easily dooe the same here without muche teachynge: for in Addition you must fyrste set downe one summe, and to the same set the other orderly, and lyke maner if you haue many: but in Subtraction you muste sette downe fyrste the greatest summe, and from it must you abate that other, euery denommation from his deweplace. S. I doe not dou bt but with a litell practise I shall attaine these bothe: but how shall I multiply and diuide after these fourmes? M. you can not duely doe none of bothe by these sortes, wherfore in suche case, you muste resort to your other artes. S. Syr, yet I se not by these sortes how to expresse hundredes, if they excede one hundred, nother yet thousandes. M. They that vse suche accomptes that exercede 100 in one summe, thei sette no 5 at the lefte hande of the scores of poundes, but thei set al the hundredes in an other farther rowe, and 500 at the left hande therof, and the thousandes thei set in a farther rowe yet, and at the lefte syde therof thei set the 5000, and in the space ouer thei sette the 10000, and in a hygher rowe 20000, whiche all I haue expressed in this exaumple, which is 57869 li 12s. 9d. ob. q. for

I had

maie receaue but one counter (as all other spaces lyke wayes doo) and that one standeth in that place for 6 d. Lyke waies betwene the shyllynges and the poundes, one counter standeth for 10 s. And betwene the poundes and 20 l. one counter standeth for 10 poundes. But beside those you maie see at the left syde of shyllynges, that one counter standeth alone, & betokeneth 5 s. So against ỹ poundes, ỹ one counter stãdeth for 5 li. And against ỹ 20 poundes, the one cõunter standeth for 5 scoze poundes, ỹ is 100 li, so ỹ euery syde cõuter is 5 tymes so much as one of thẽ against which he standeth. Now for ỹ accoumpt of auditozs take this exãple.

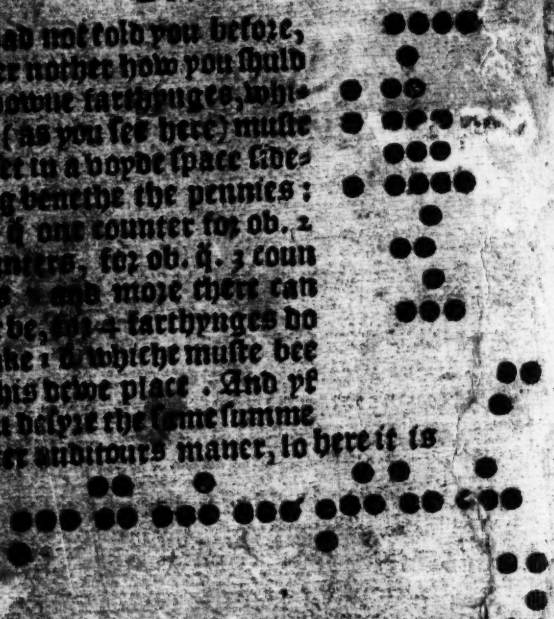
Where I haue expzessed the same sũme 198 li. 19 s. 11 d. But here you see the pennies stande towarde the right hande, and the other encreasynge orderly towarde the lefte hande. Againe you maie see, that auditozs will make lynes (yea and moze) for pennies, shillynges, and all other valewes, if theyr summes extende therto. Also you see, that thei set one counter at the right ende of eche rowe, which so set, there standeth for 5 of that roume: and on the lefte cozner of the

rowe

rowe it standeth for 10 of the same rowe. But now if you wolde adde other subtracte after any of bothe those sortes, if you marke the order of the other feate which I taught you you maie easily dooe the same here without muche teachynge: for in Addition you must fyrste set downe one summe, and to the same set the other orderly, and lyke maner if you haue many: but in Subtraction you muste sette downe fyrste the greatest summe, and from it must you abate that other, euery denommation from his dewe place. S. I doe not dou bt but with a littell practise I shall attaine these bothe: but how shall I multiply and diuide after these fourmes? M. you can not duely doe none of bothe by these sortes, wherfore in suche case, you muste resort to your other artes. S. Syr, yet I se not by these sortes how to expresse hundredes, if they excede one hundred, nother yet thousandes. M. They that vse suche accomptes that it excede 200 in one summe, thei sette no 5 at the lefte hande of the scores of pounde, but thei set al the hundredes in an other farther rowe, and 500 at the left hande therof, and the thousandes thei set in a farther rowe yet, and at the lefte syde therof thei set the 5000, and in the space ouer thei sette the 10000, and in a hygher rowe 20000, whiche all I haue expressed in this exaumple, which is 57869 li 12 s. 9 d. ob. q. for

I had

I had not told you before, wher nother how you shuld set downe farthynges, whiche (as you see here) muste be set in a voyde space sidelyng benethe the pennies: for q̃ one counter for ob. 2 counters, for ob. q̃. 3 counters: and more there can not be, for 4 farthynges do make 1 ꝺ whiche muste bee in his dewe place. And yf you desyre the same summe after auditours maner, lo here it is



But in this thyng you shal take this for sufficient, and the rest you shall obserue as you maie see by the workyng of eche sorte: for the dyuers wittes of men haue inuented diuers and sundry waies almost vnnumerabl.

But one feate I shall teache you, whiche not onely for the strangenesse and secretenes is muche pleasaunt, but also for the good cōmoditee of it, ryght worthy to be well marked. This feate hath been vsed aboue 2000 yeres at the least, and yet was it neuer commonly knowen, especially in Englysshe it was